Comparing Proprioceptive Acuity in the Arm between Joint Space and Task Space

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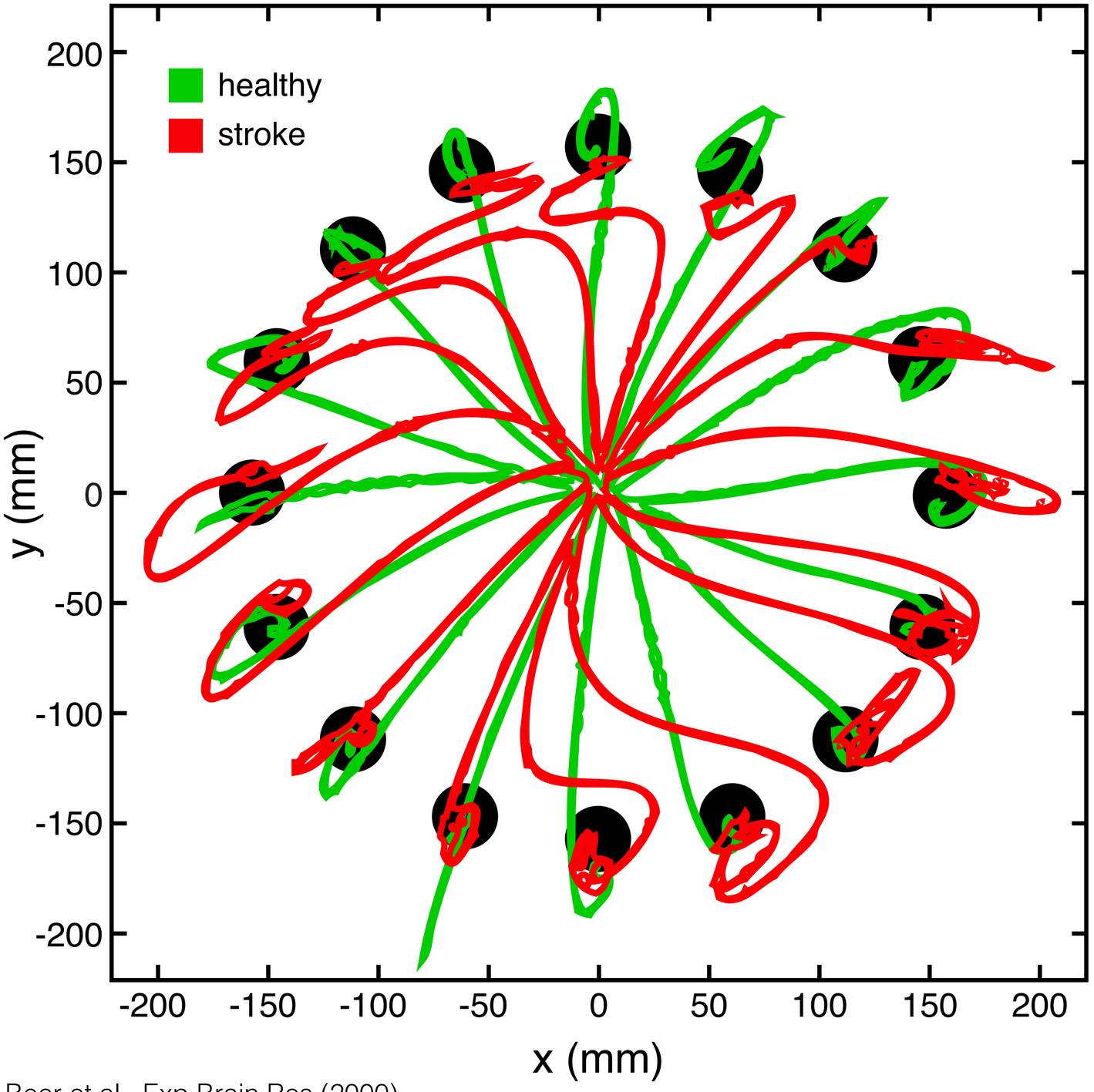
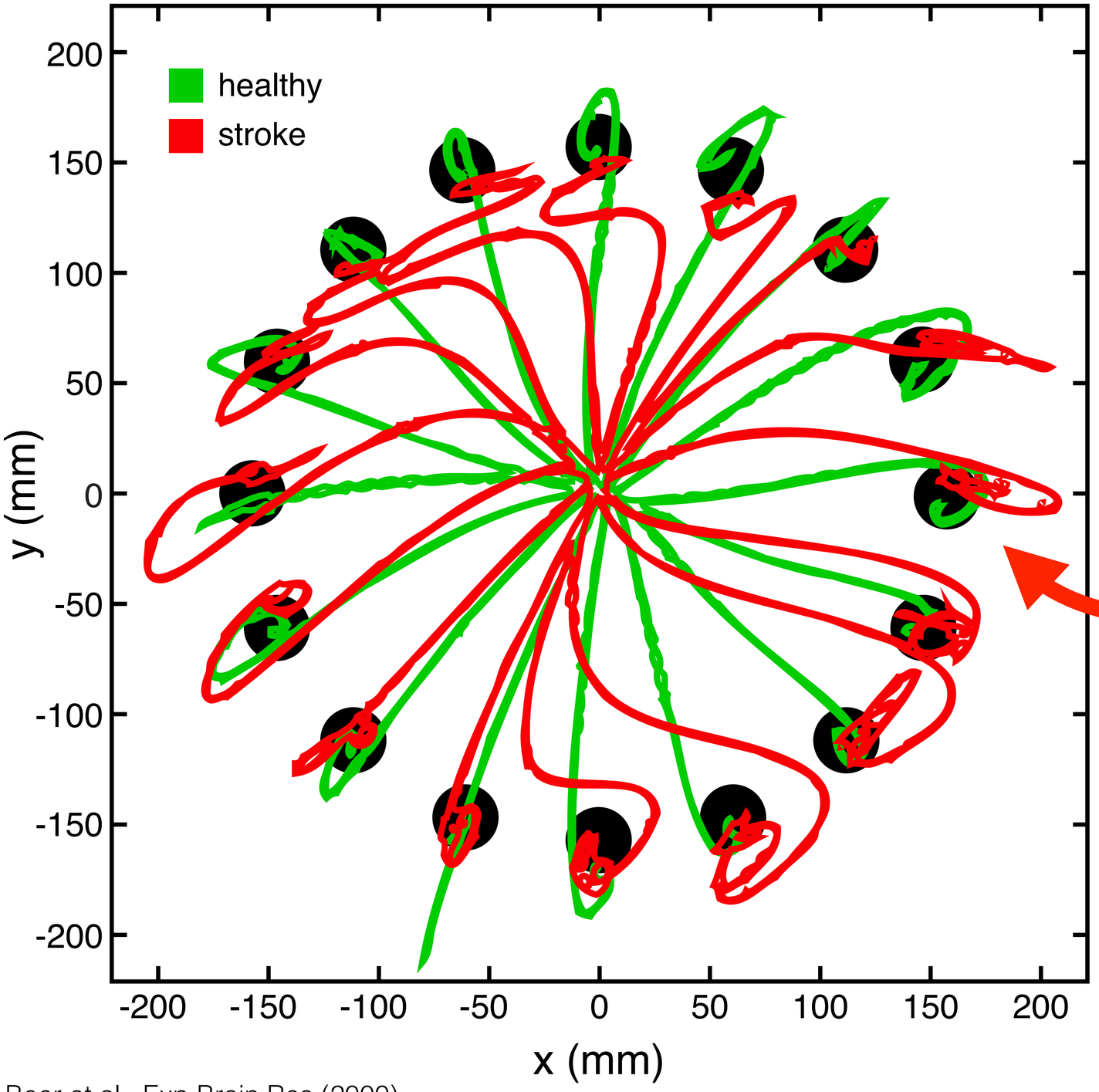
charmlab

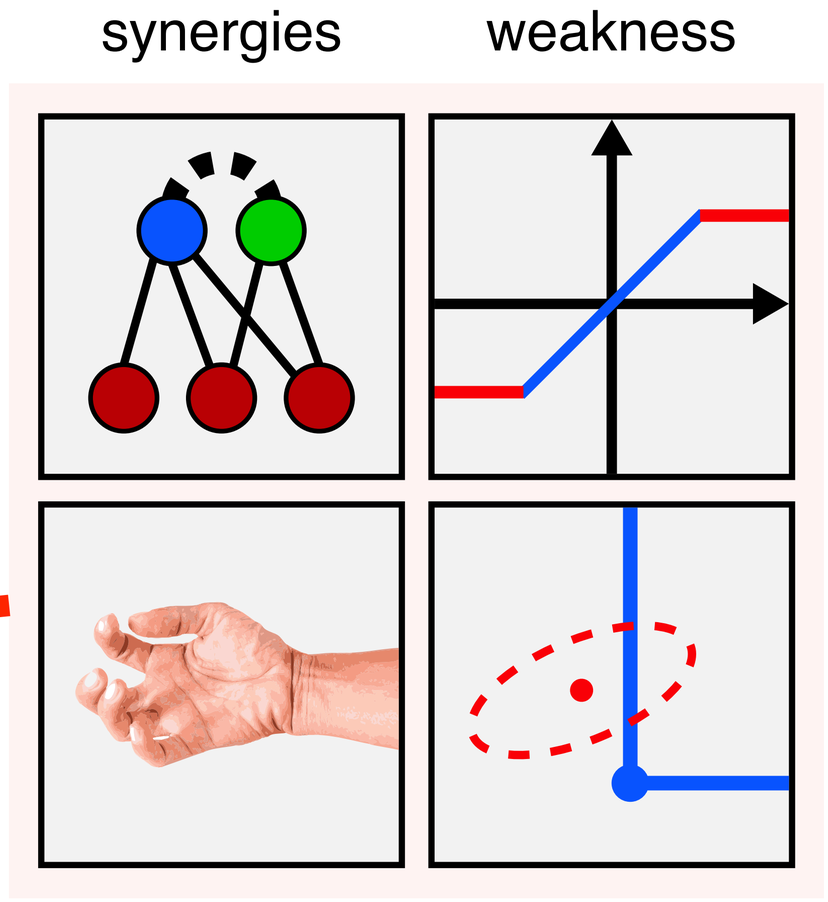


Beer et al., Exp Brain Res (2000)

How do we move?

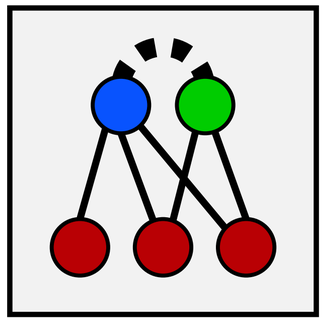


Beer et al., Exp Brain Res (2000)

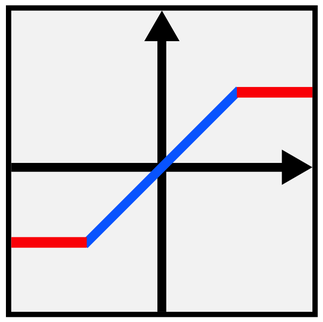


Dewald et al., Brain (1995)
Bourbonnais & Noven, Am J Occup Ther (1989)
Watkins et al., Clin Rehabil (2002)
Cusmano et al., J Healthcare Eng (2014)

synergies
&
weakness



+



spasticity

Task selection

Optimal
feedback
control law

Motor
commands

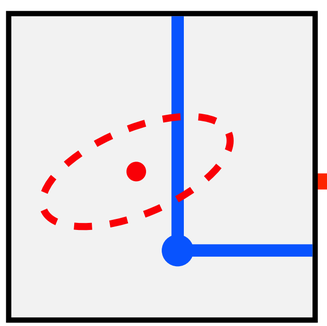
Movement

Efferent
copy

System state
(positions, velocities,
forces)

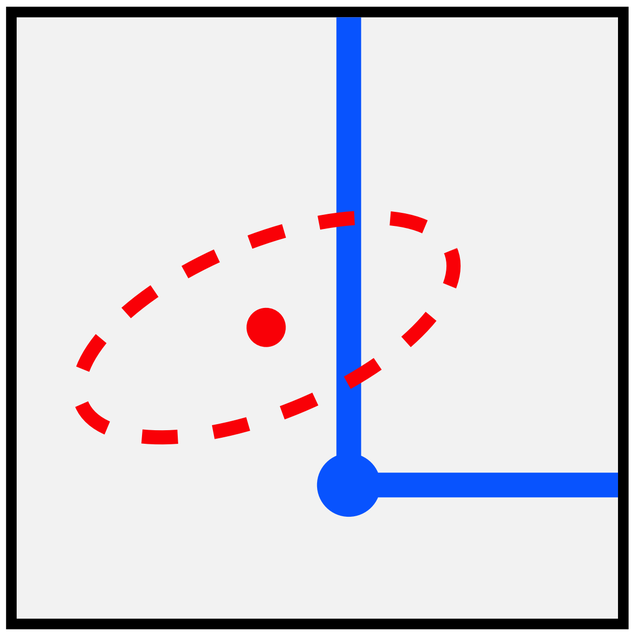
Optimal
state estimator

Sensory
feedback



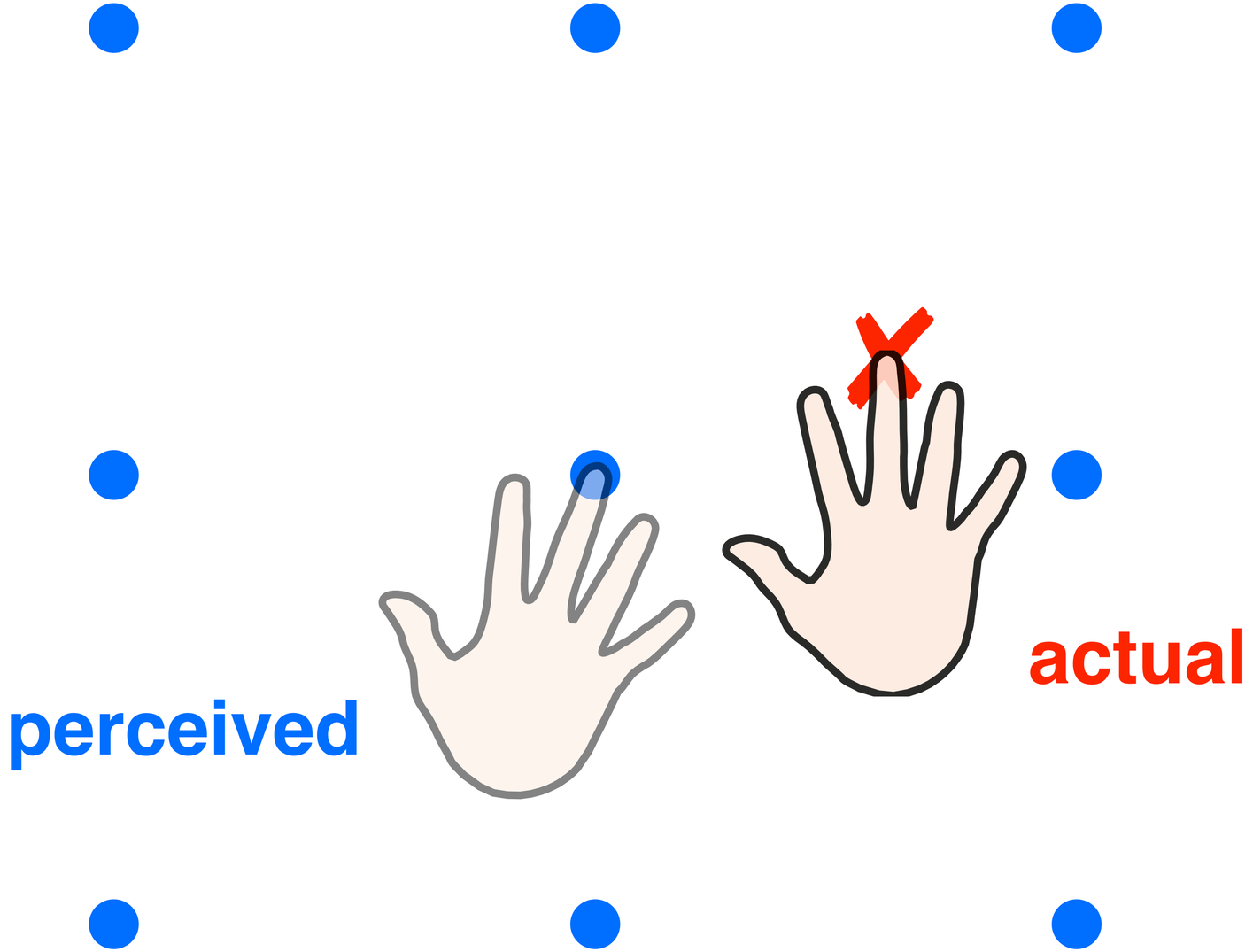
sensory
uncertainty

Scott, Nature Rev Neurosci (2004)
Todorov & Jordan, Nature Neurosci (2002)
Todorov, Nature Neurosci (2004)
Beer et al., Muscle and Nerve (2007)
Zackowski et al., Brain (2004)
Wagner et al., Neurorehabil Neural Repair (2006)



What is the relationship between proprioception as quantified in **task-relevant space** and as measured at **individual joints**?

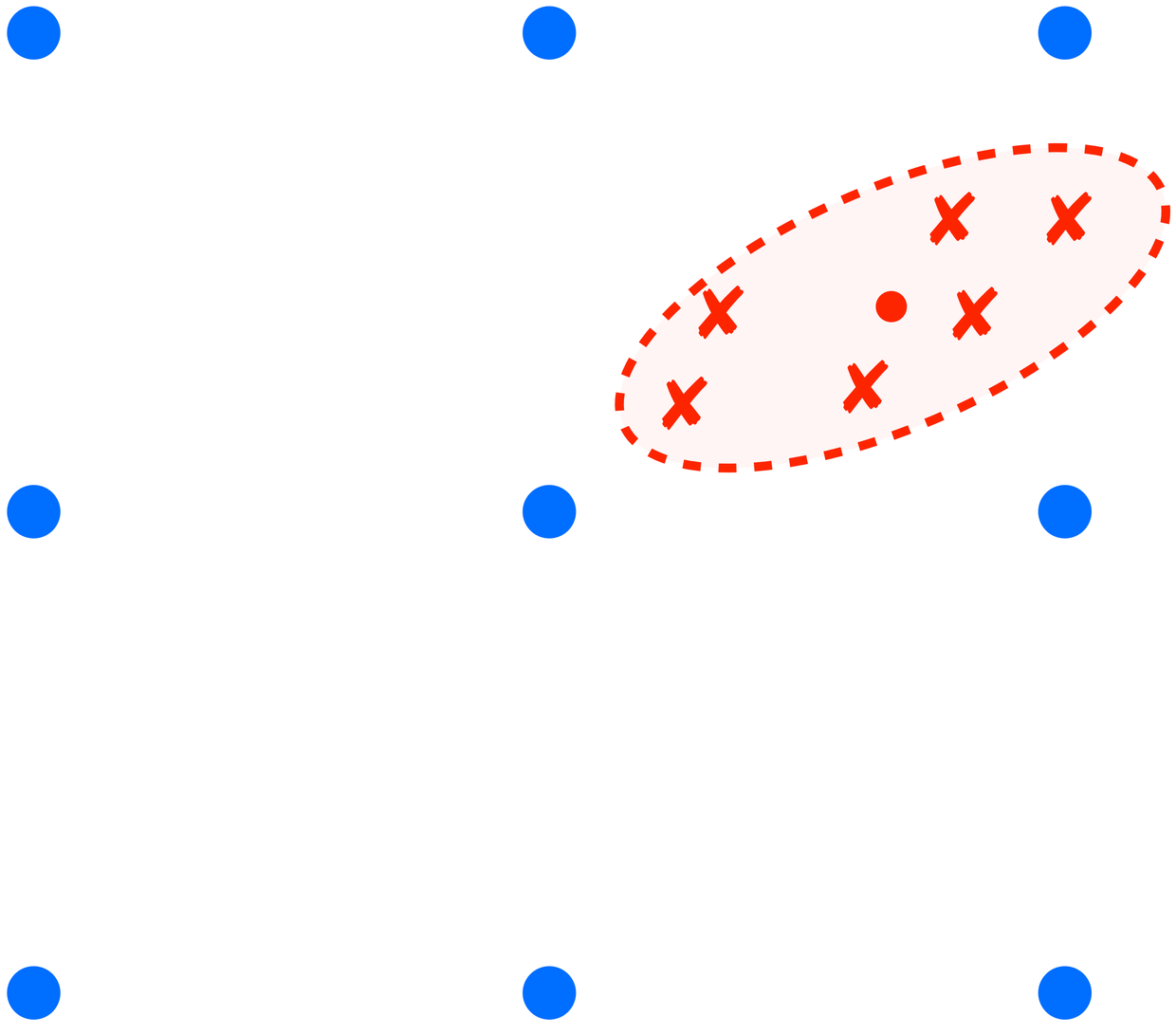
task space



perceived

actual

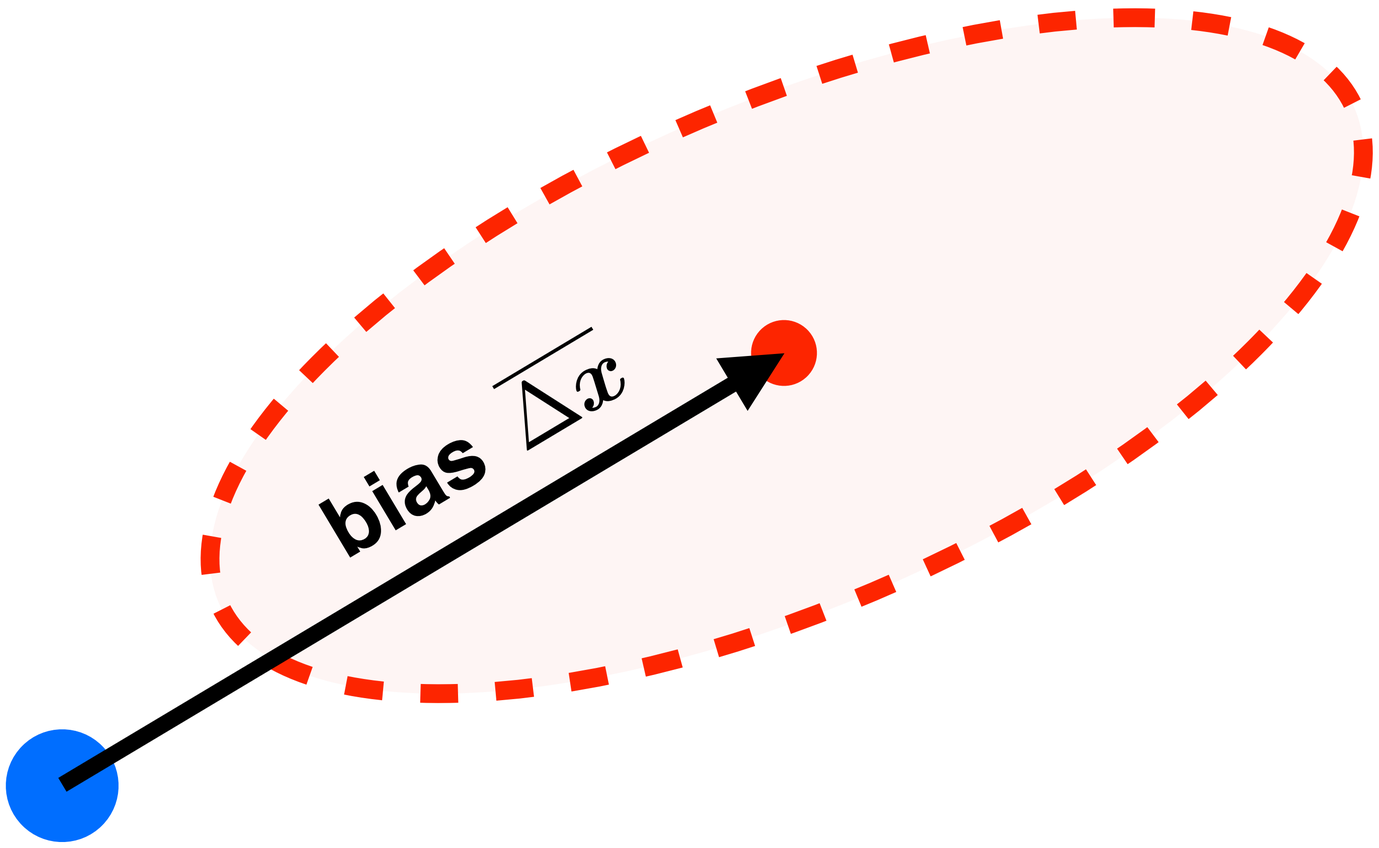
task space



task space

Dukelow et al., Neurorehabil Neural Repair (1995)
Gordon et al., J Neurophysiol (1995)
Cusmano et al., J Healthcare Eng (2014)
Rincon-Gonzalez et al., PLOS One (2011)

task space



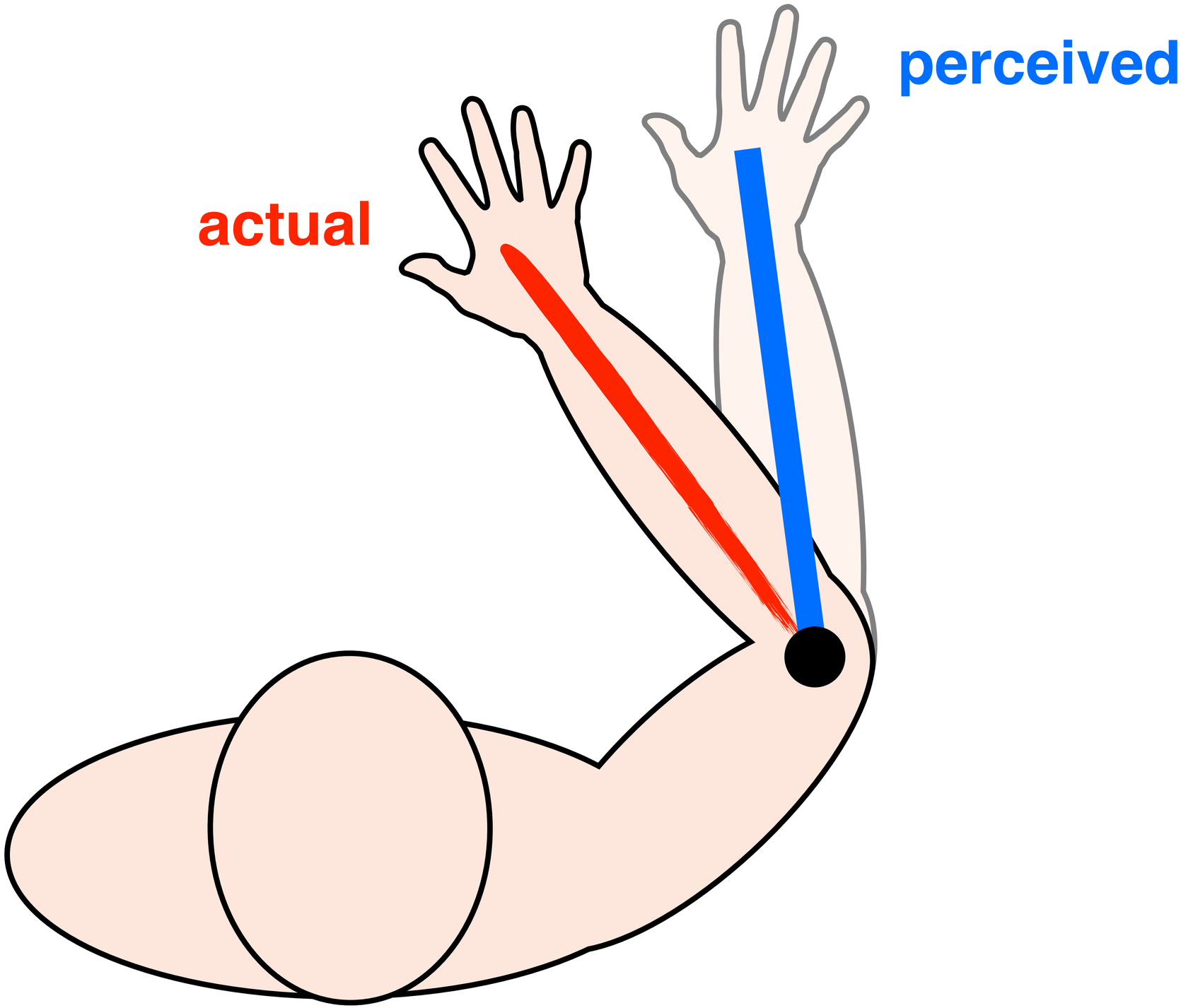
precision

$$\Sigma_{\boldsymbol{x}} = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_{yy} \end{bmatrix}$$

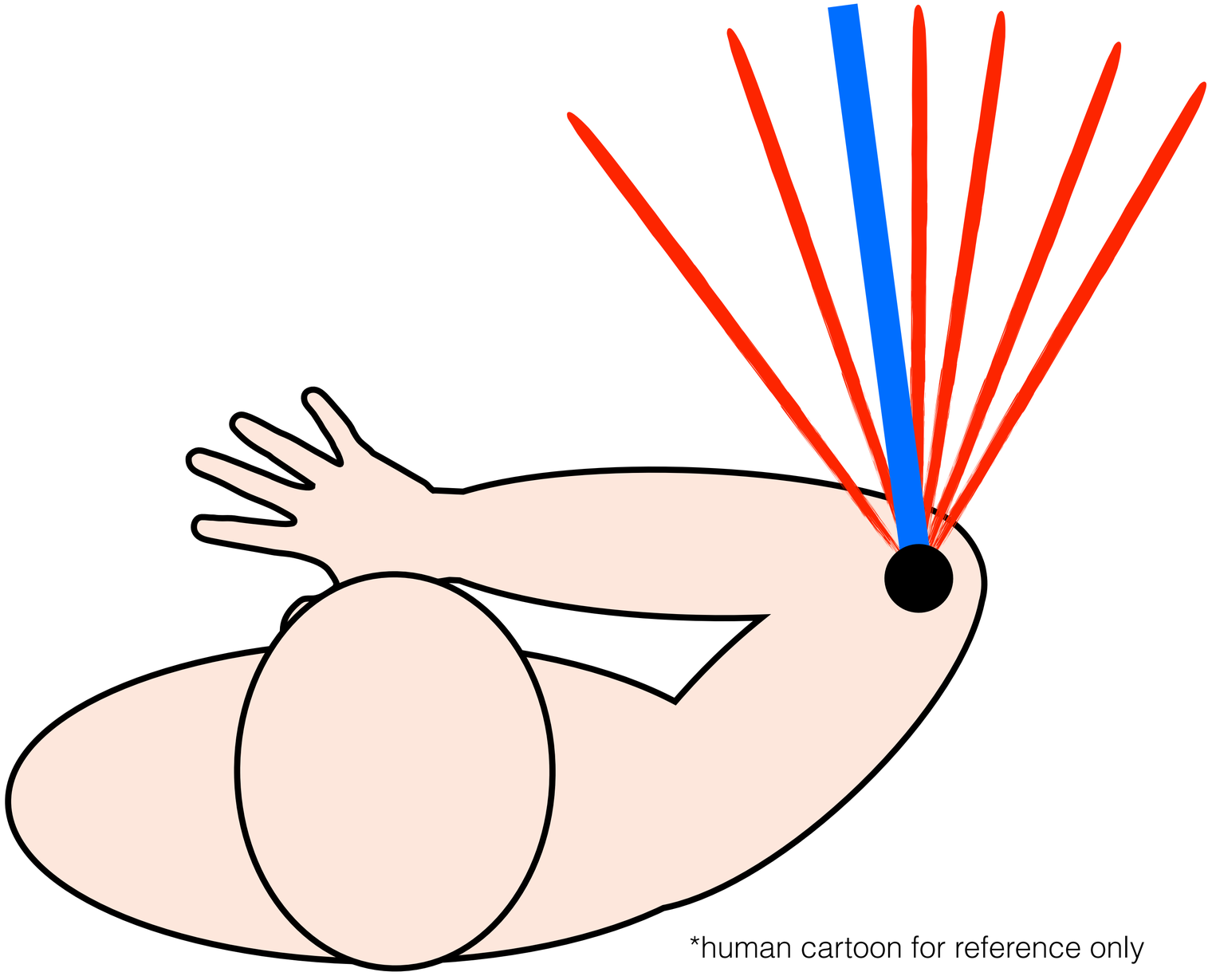
We define **proprioceptive acuity** as the functional combination of bias & precision statistics.

$$\textbf{acuity}_{\substack{\textbf{task} \\ \textbf{space}}} = h\left(\overline{\Delta \boldsymbol{x}}, \Sigma_{\boldsymbol{x}}\right)$$

joint space

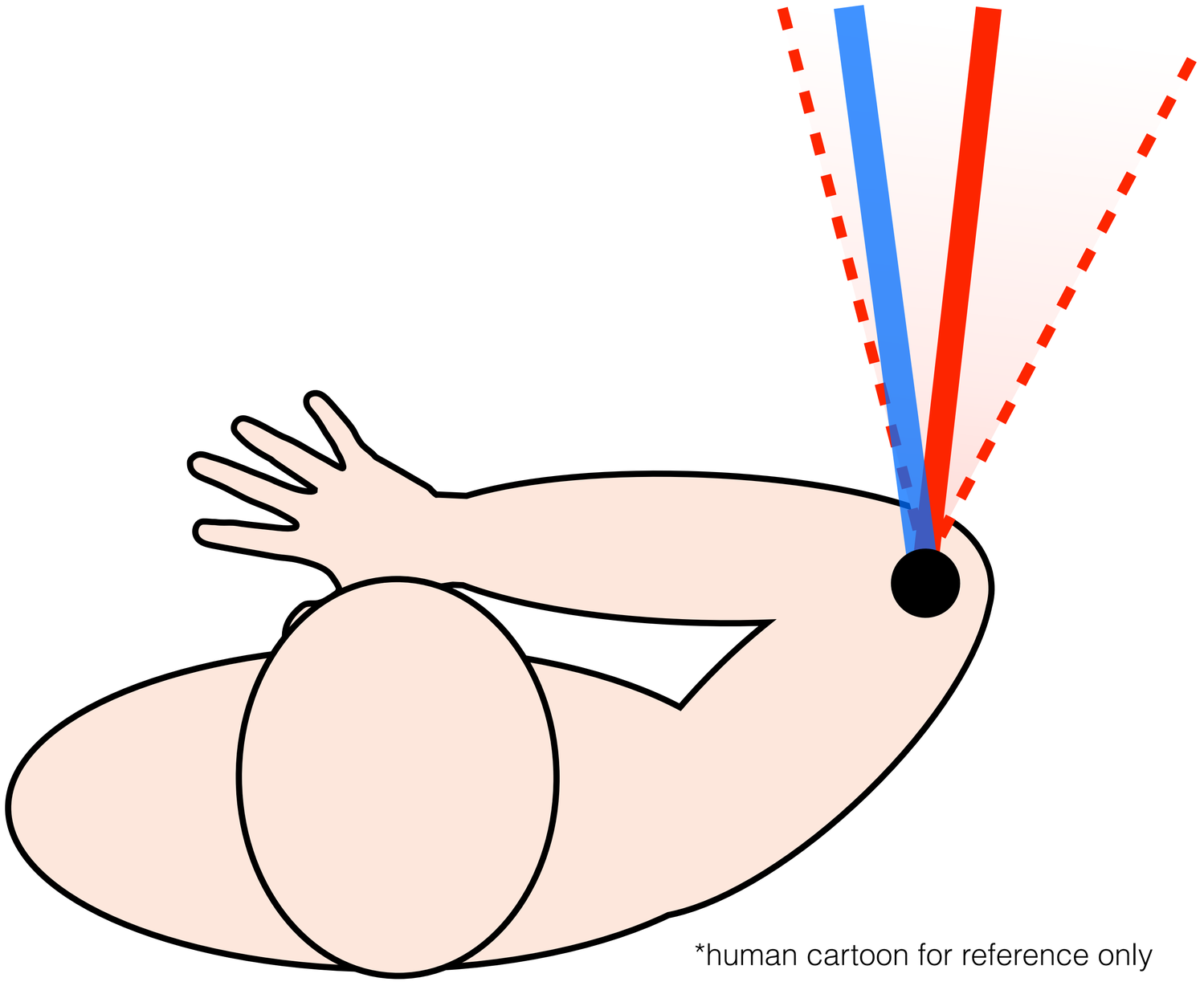


joint space



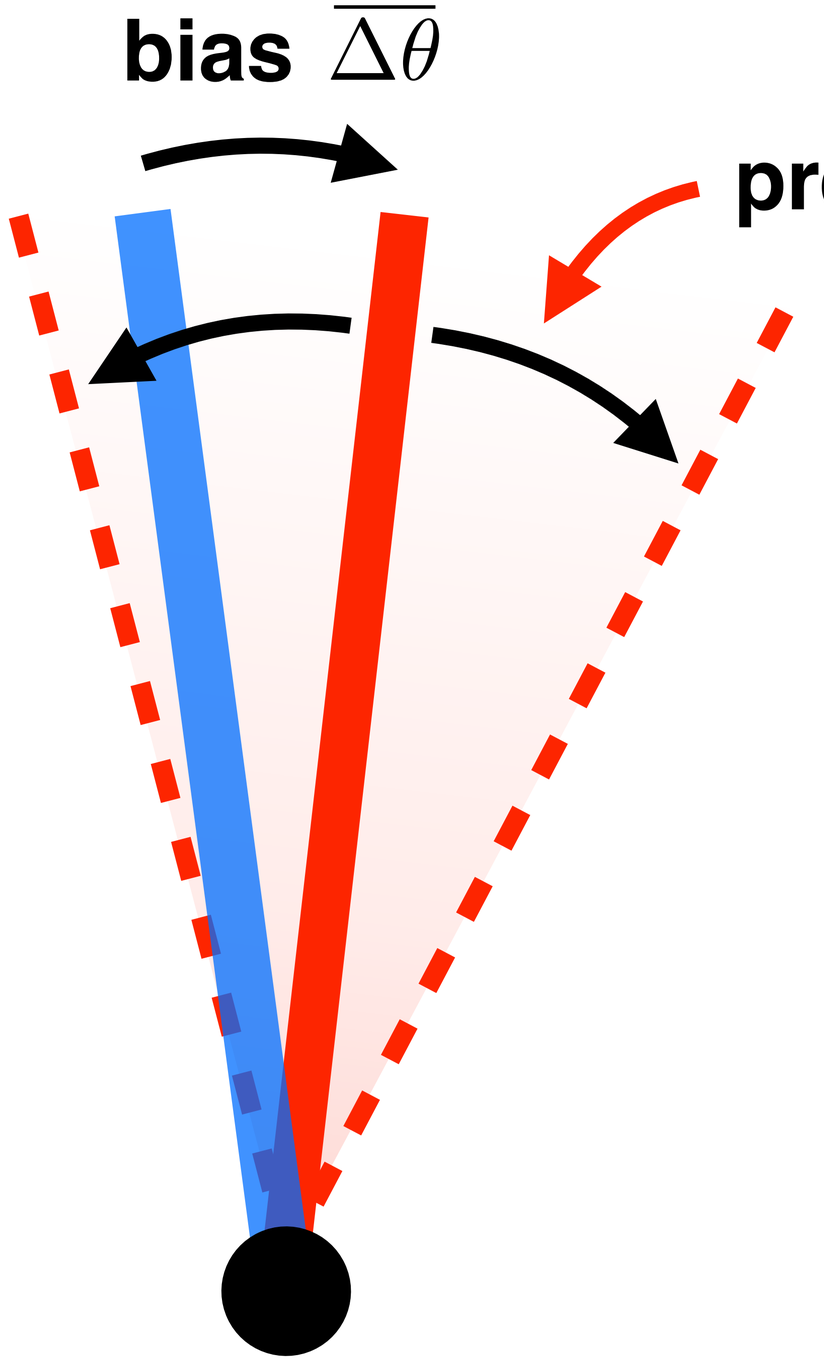
*human cartoon for reference only

joint space



*human cartoon for reference only

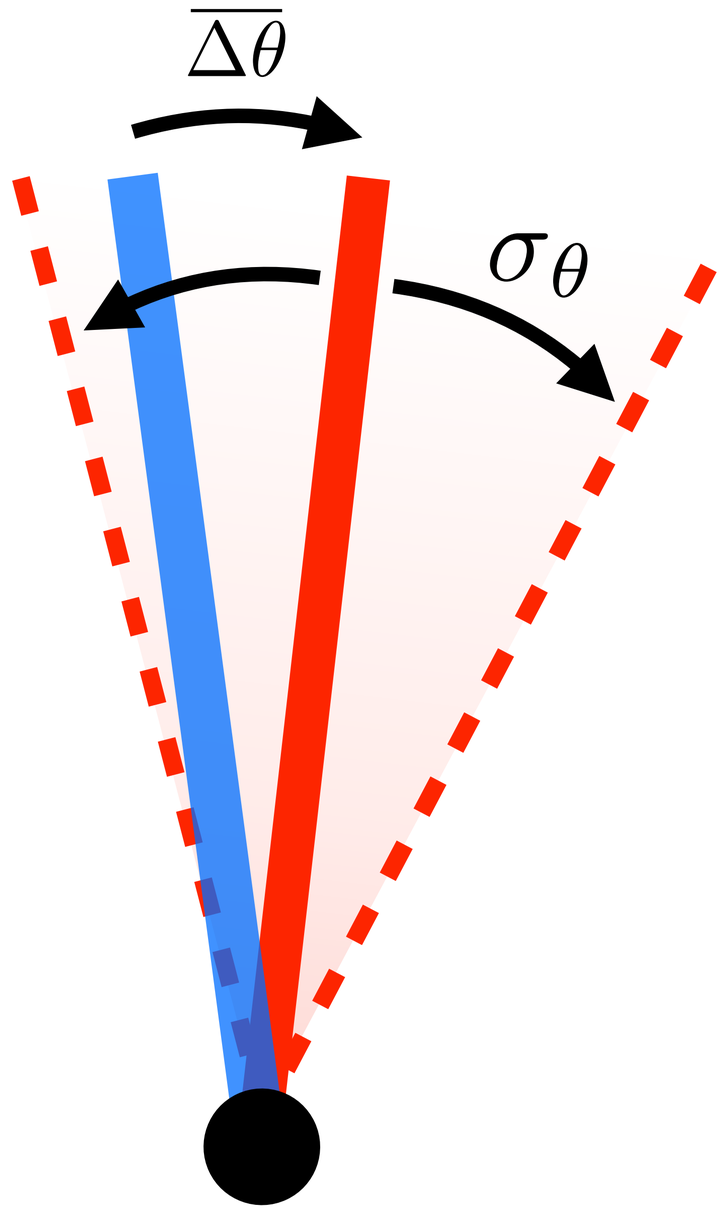
joint space



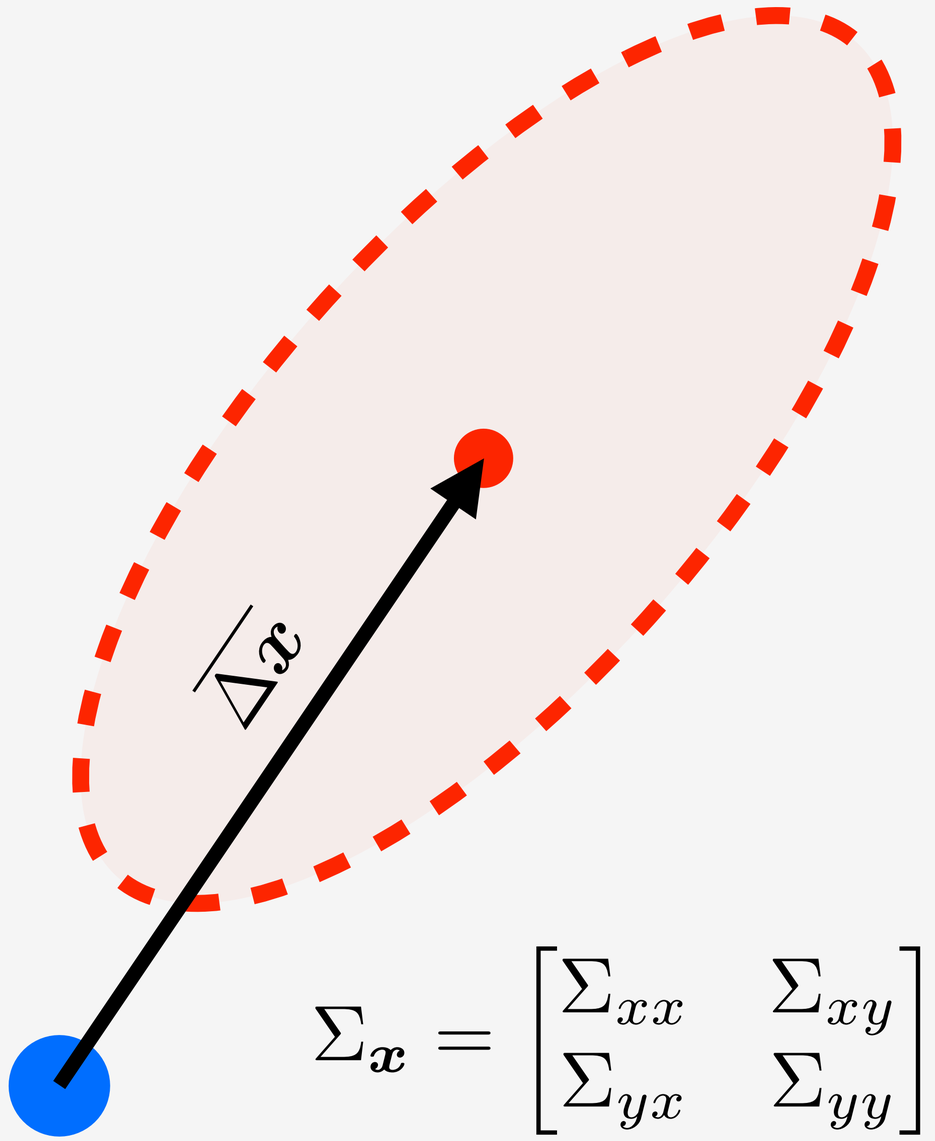
$$\textbf{acuity}_{\substack{\textbf{joint}\\\textbf{space}}} = g\left(\overline{\Delta\theta}, \sigma_\theta\right)$$

Karduna & Sainberg., PLOS One (2012)
King et al., J Mot Behav (2013)
Fuentes & Bastian, J Neurophysiol (2010)
Aydin et al., Arch Orthop Trauma Surg (2001)
Chapman et al., J Orthop Res (2009)
Hall & McCloskey, J Physiol (1983)
Scott & Loeb, J Neurosci (1994)

joint space

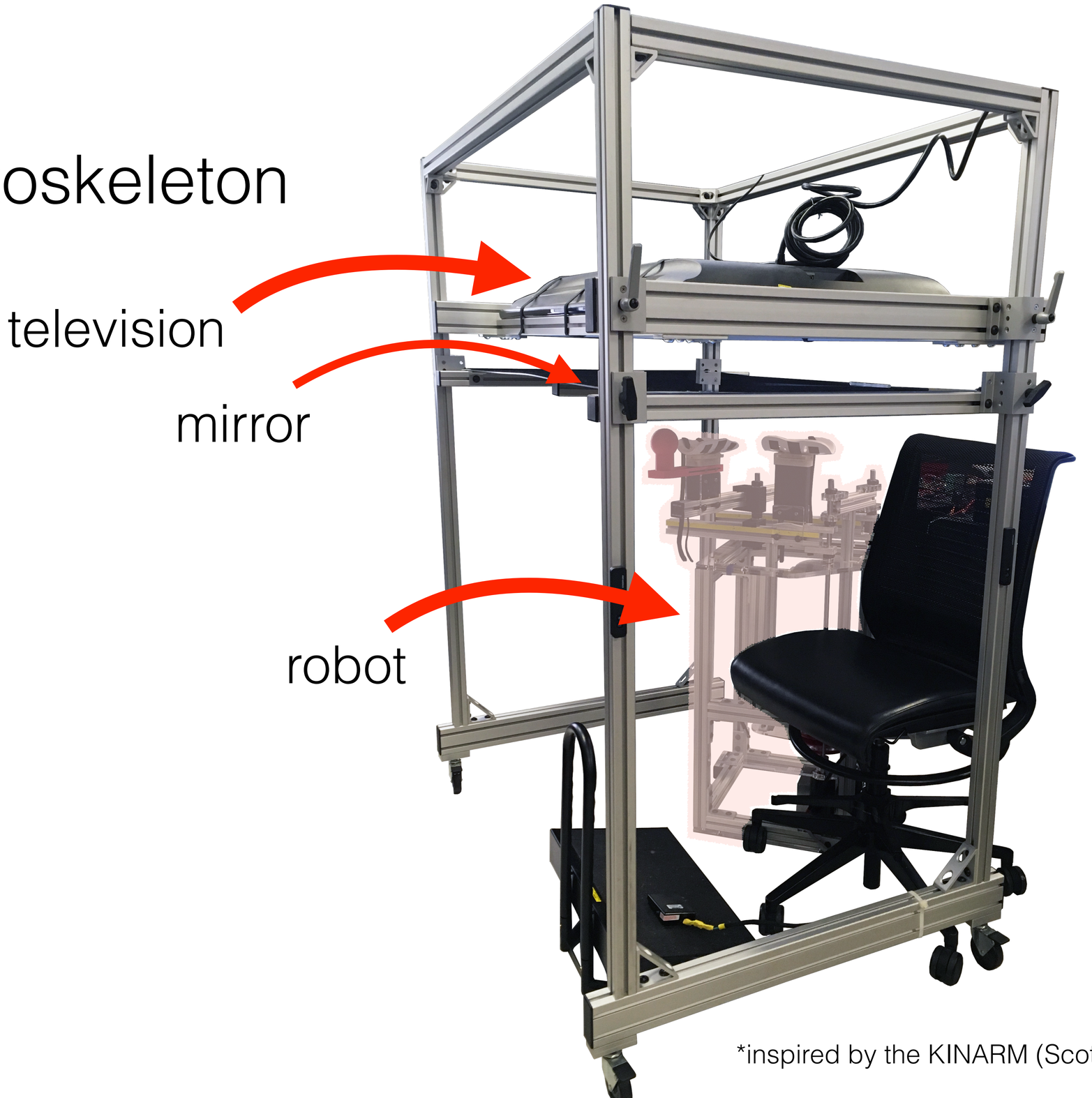


task space



chARM

2-DOF planar exoskeleton



*inspired by the KINARM (Scott,1999)

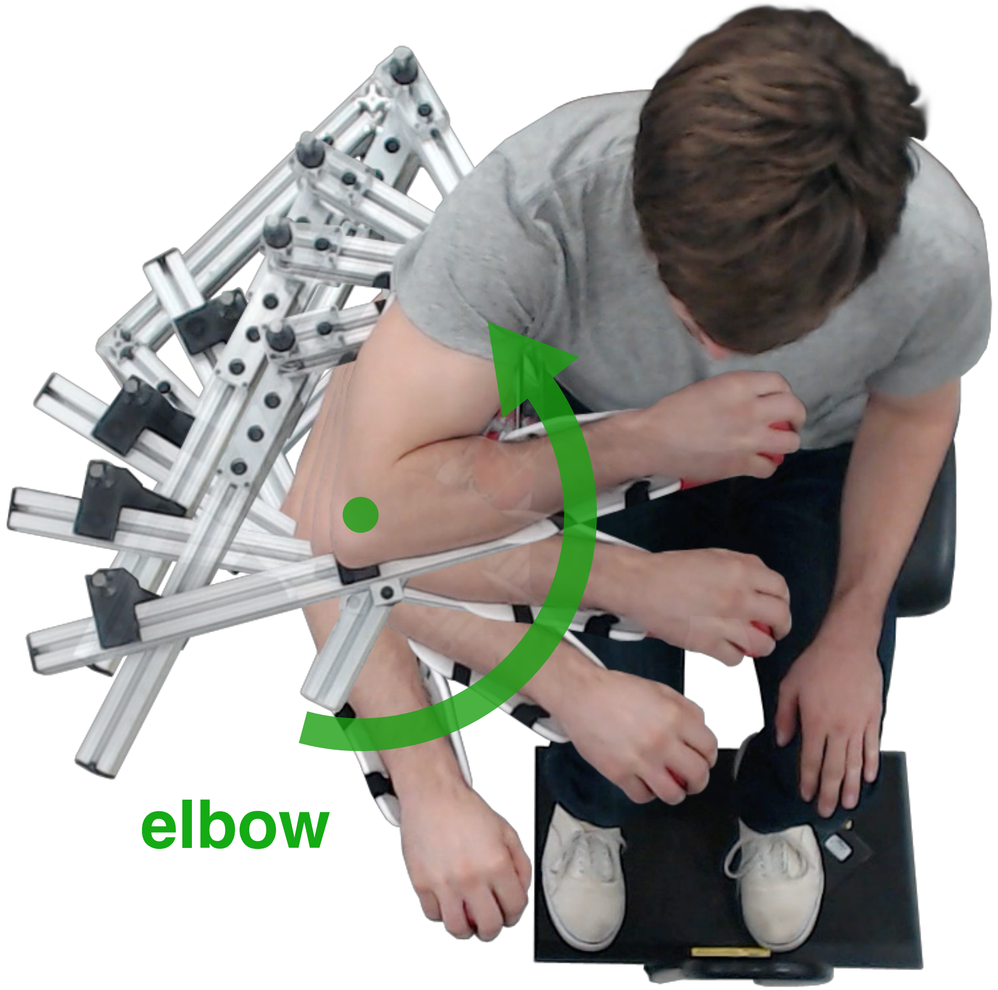
chARM

2-DOF planar exoskeleton

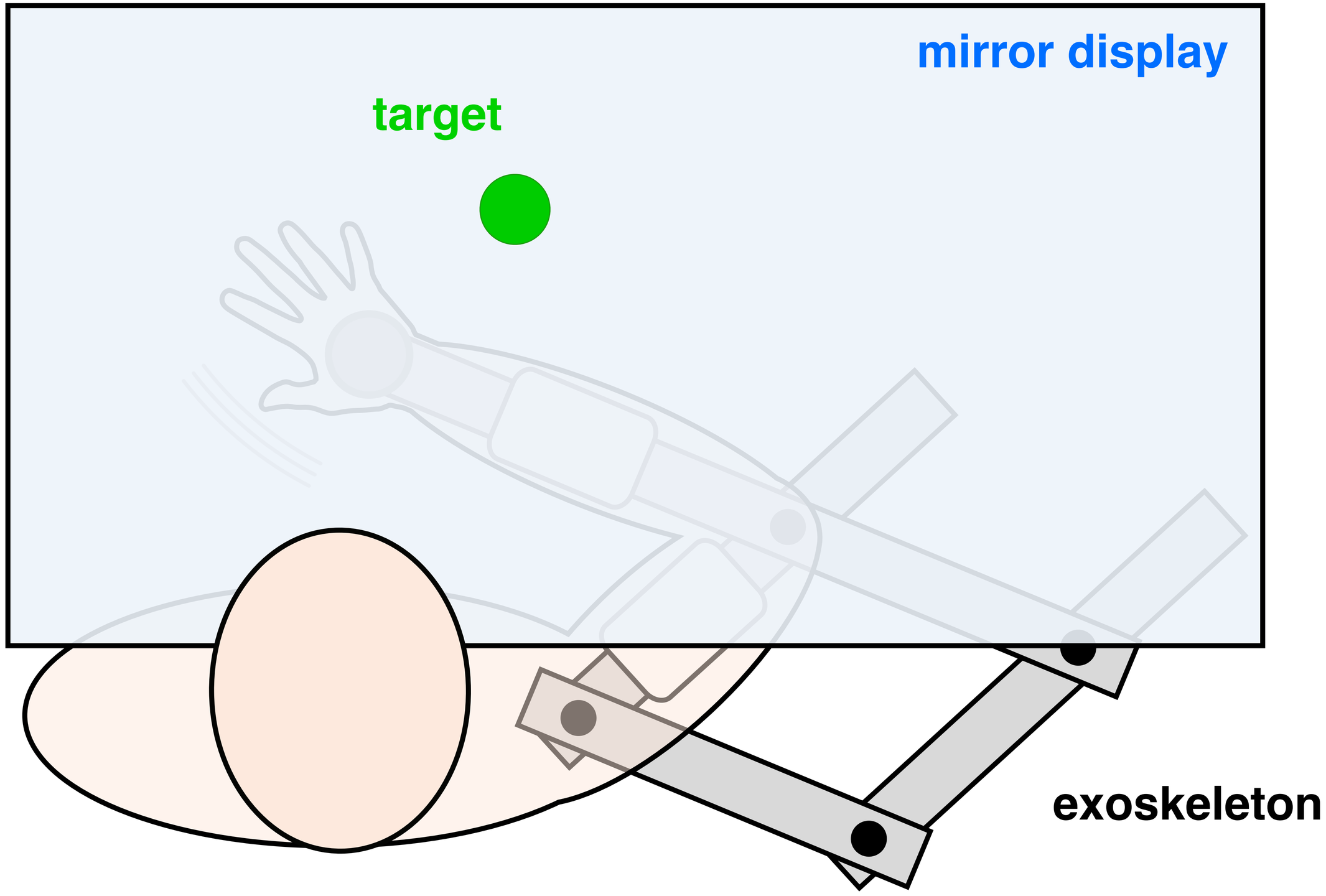
*inspired by the KINARM (Scott,1999)

chARM

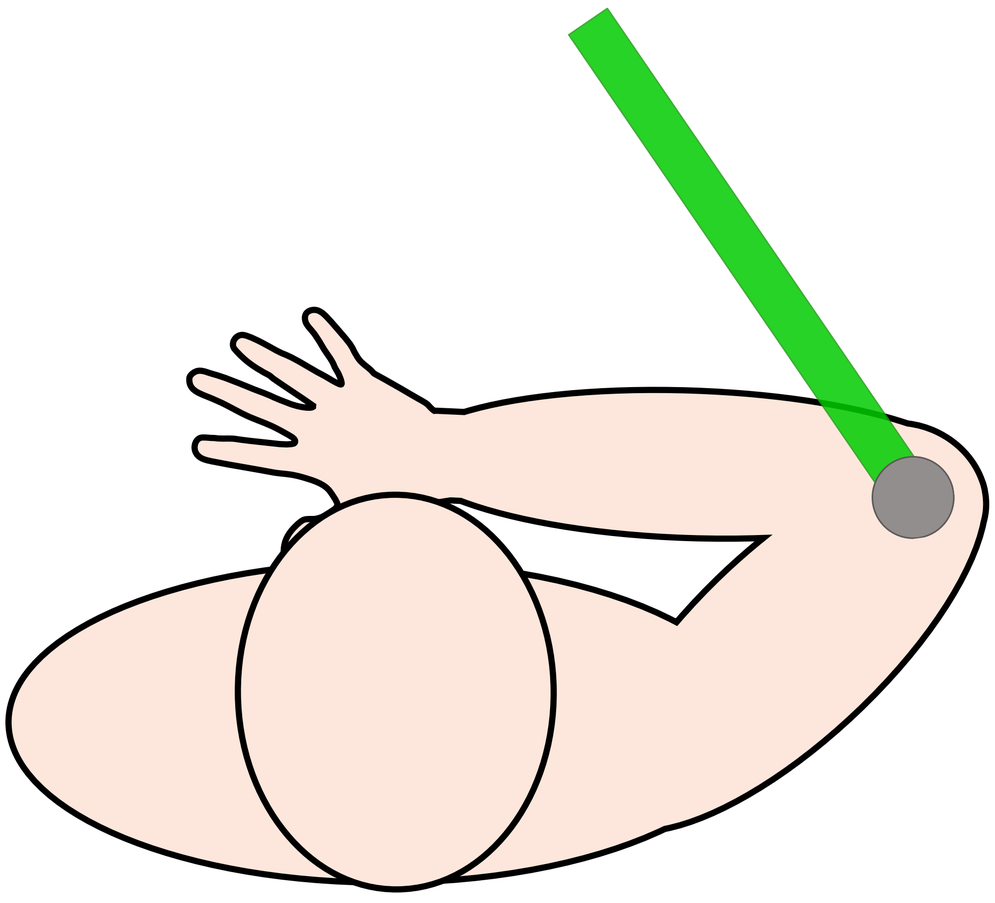
2-DOF planar exoskeleton



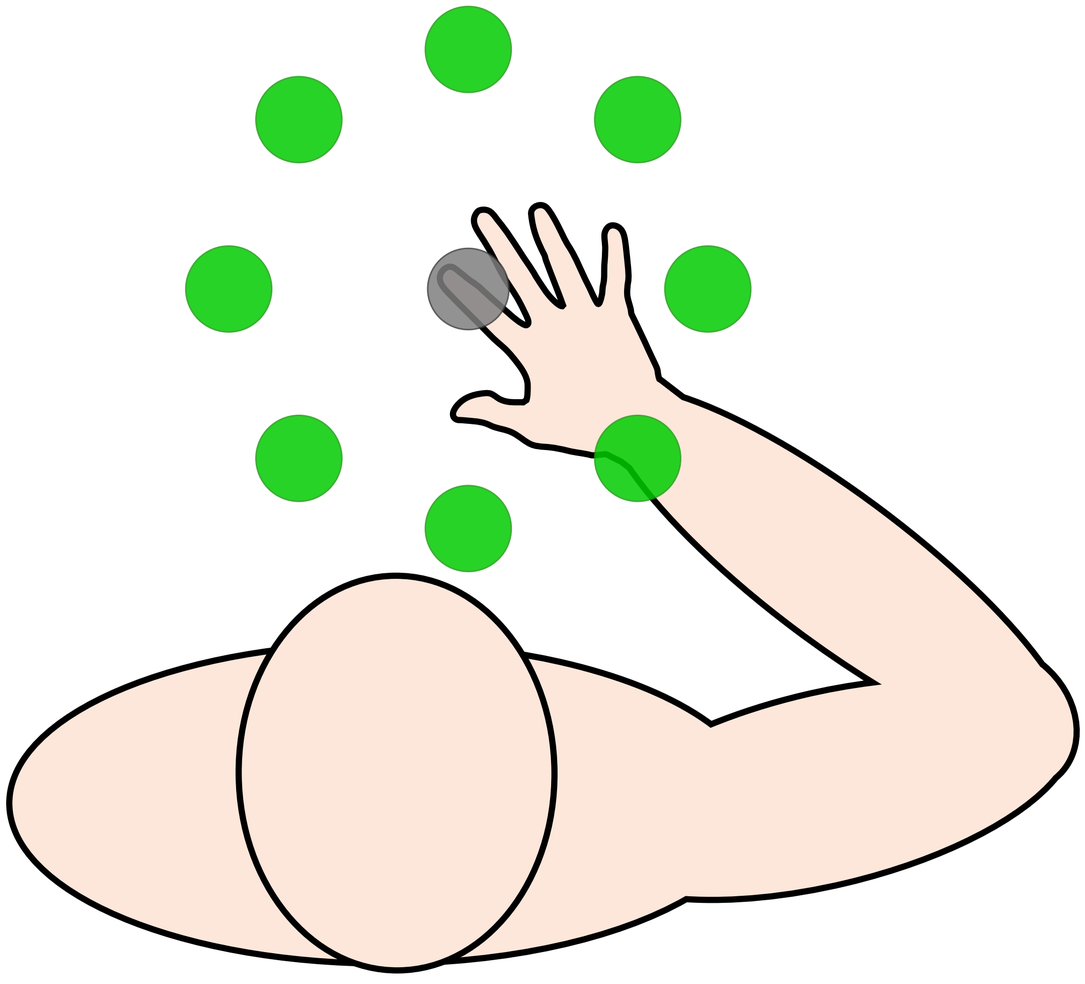
*inspired by the KINARM (Scott,1999)


mirror display
target
exoskeleton

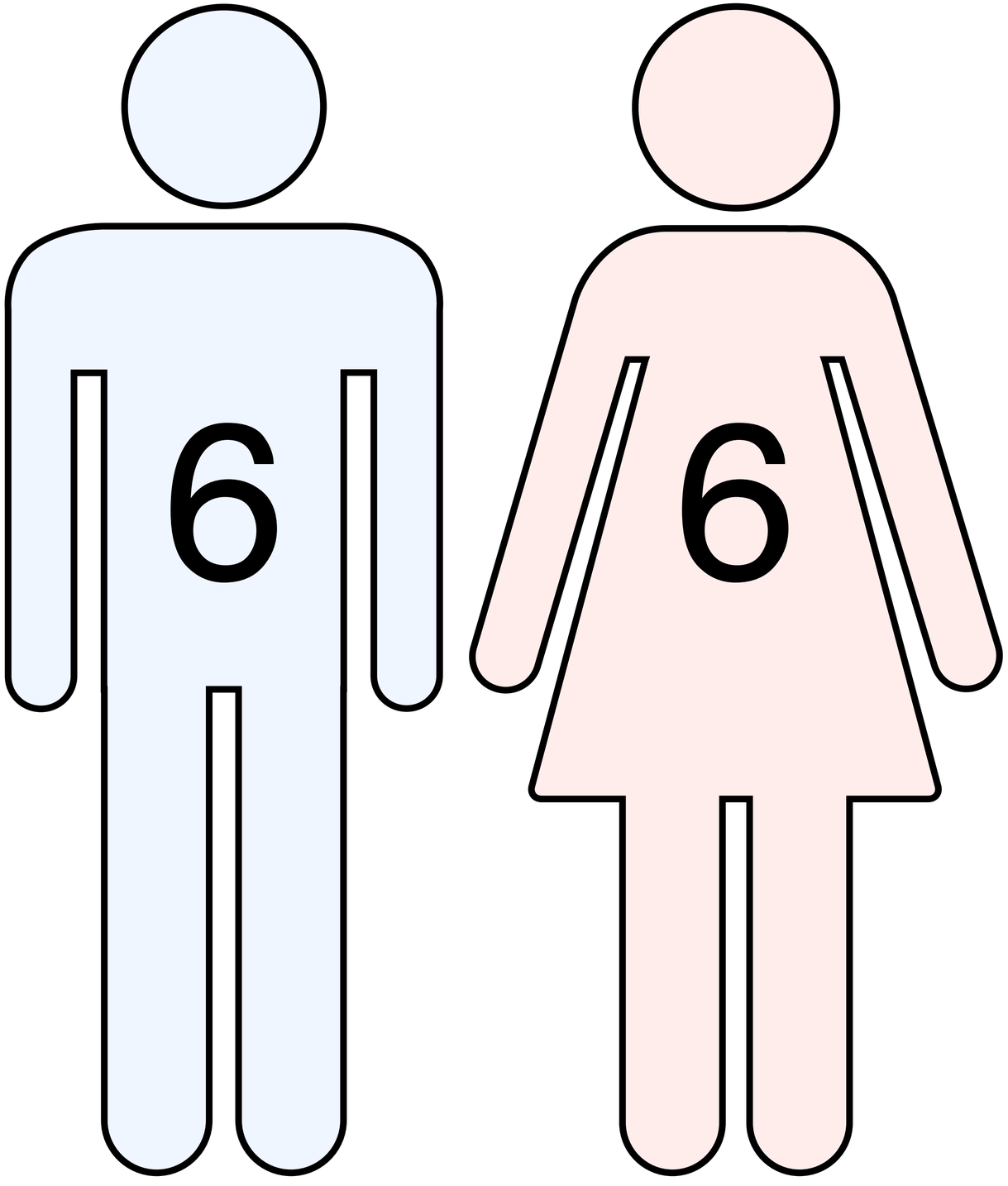
joint space



task space



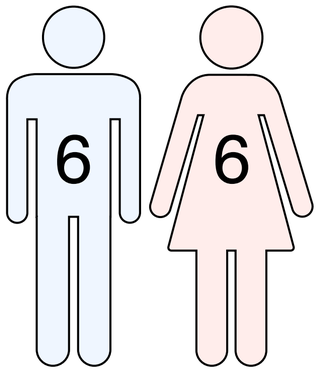
*exoskeleton & display removed for clarity

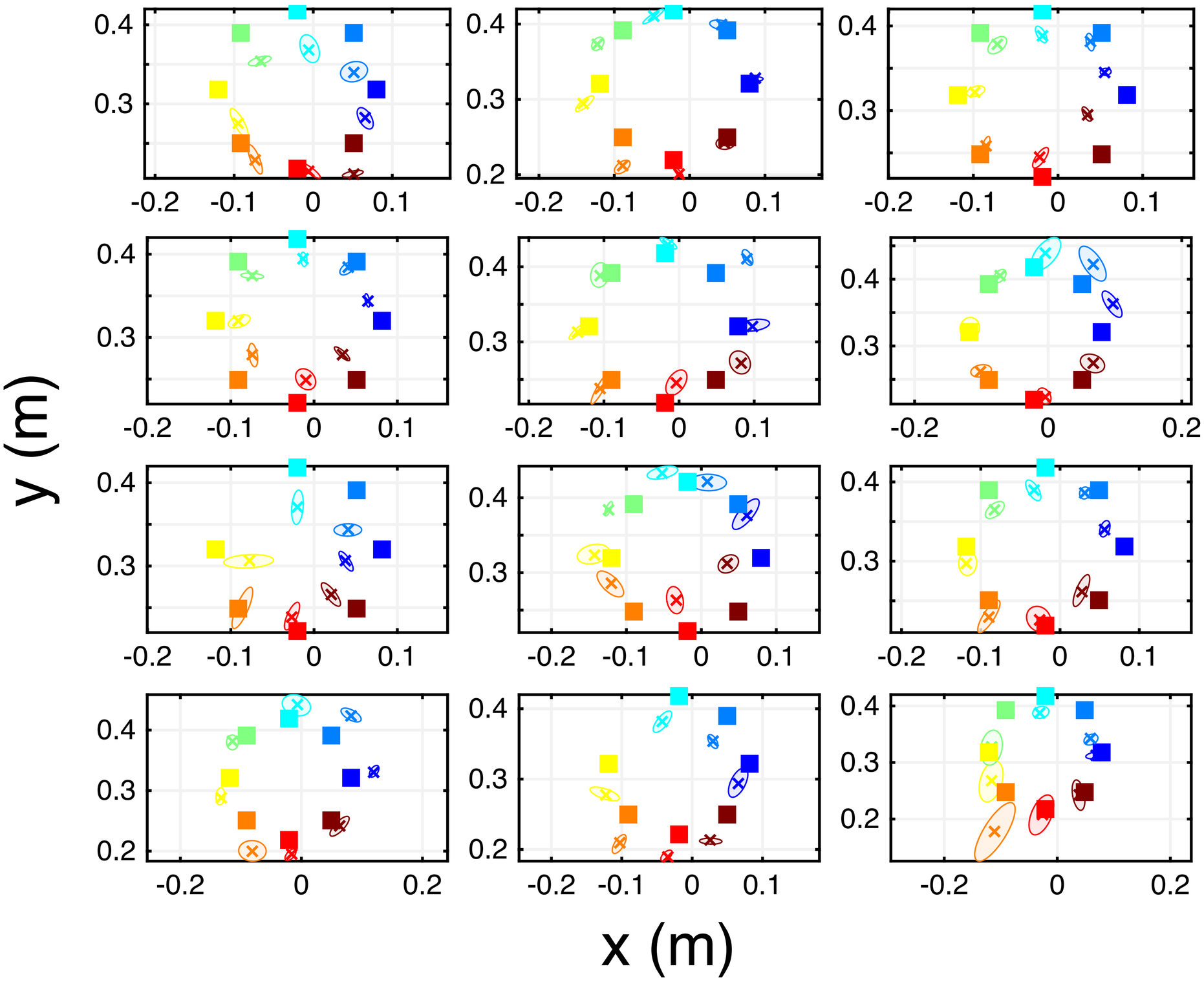


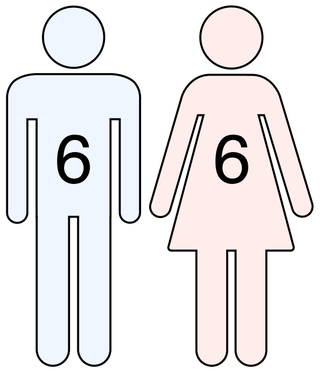
participants

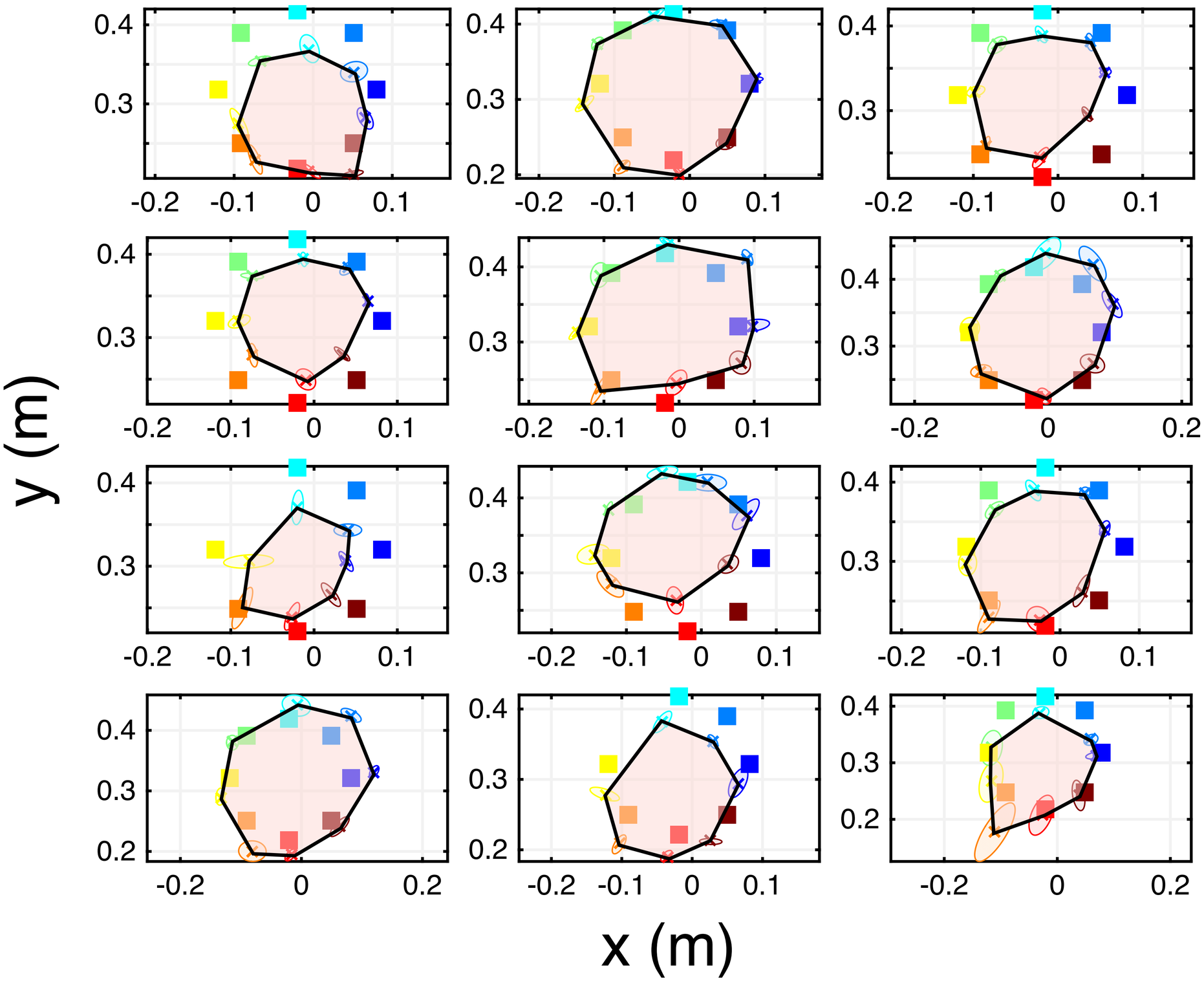
25 ± 5 yrs

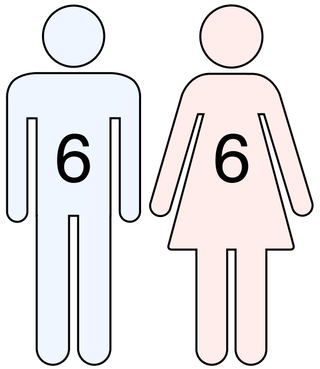
right-handed

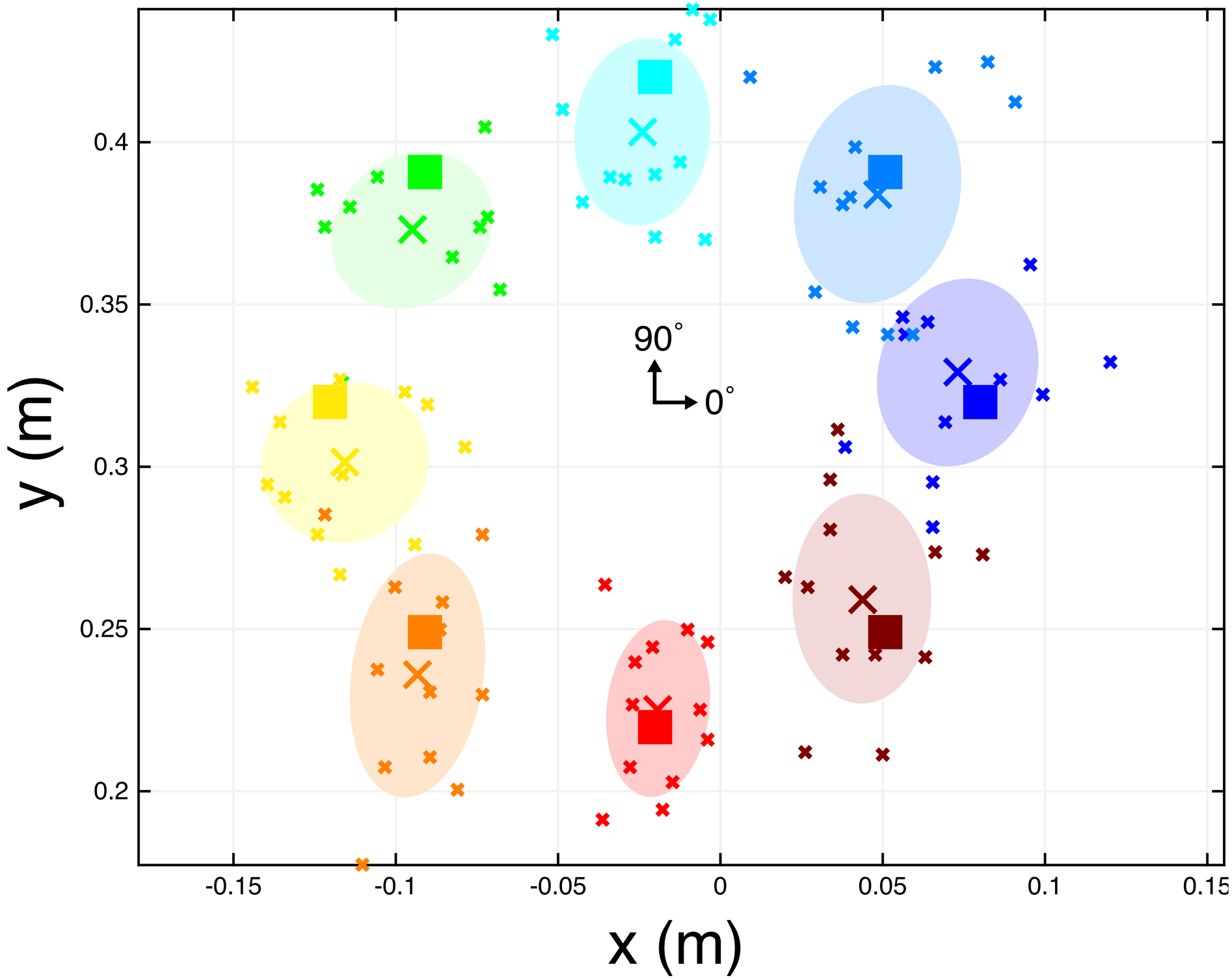

6
6

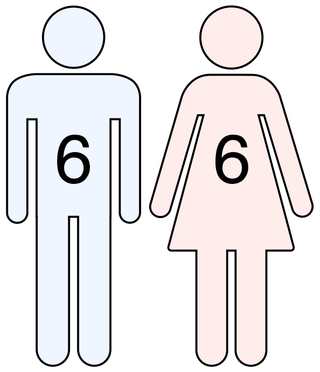

y (m)
x (m)
0.4
0.3
0.2
-0.2
-0.1
0
0.1
0.2

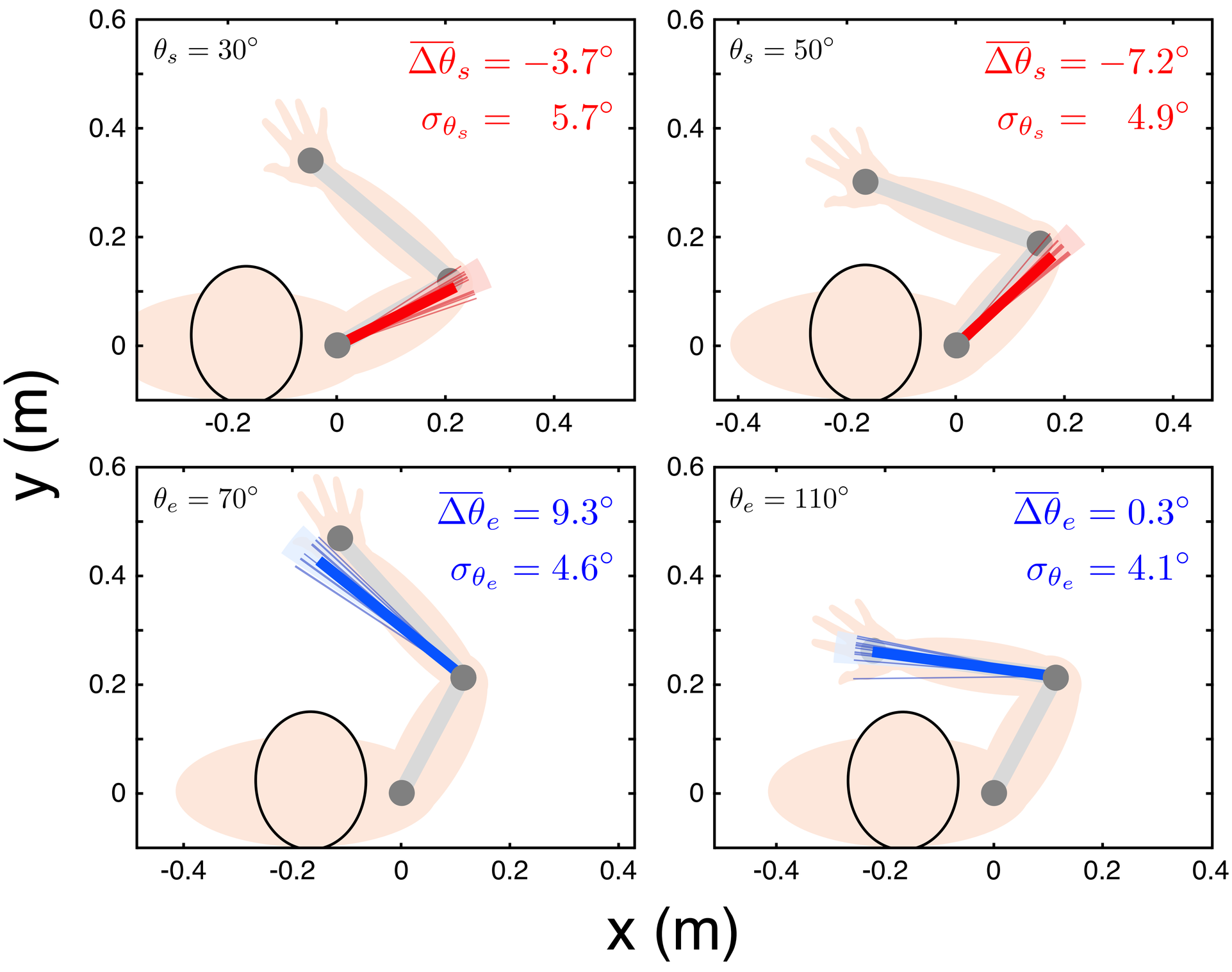

6
6


y (m)
x (m)


6
6

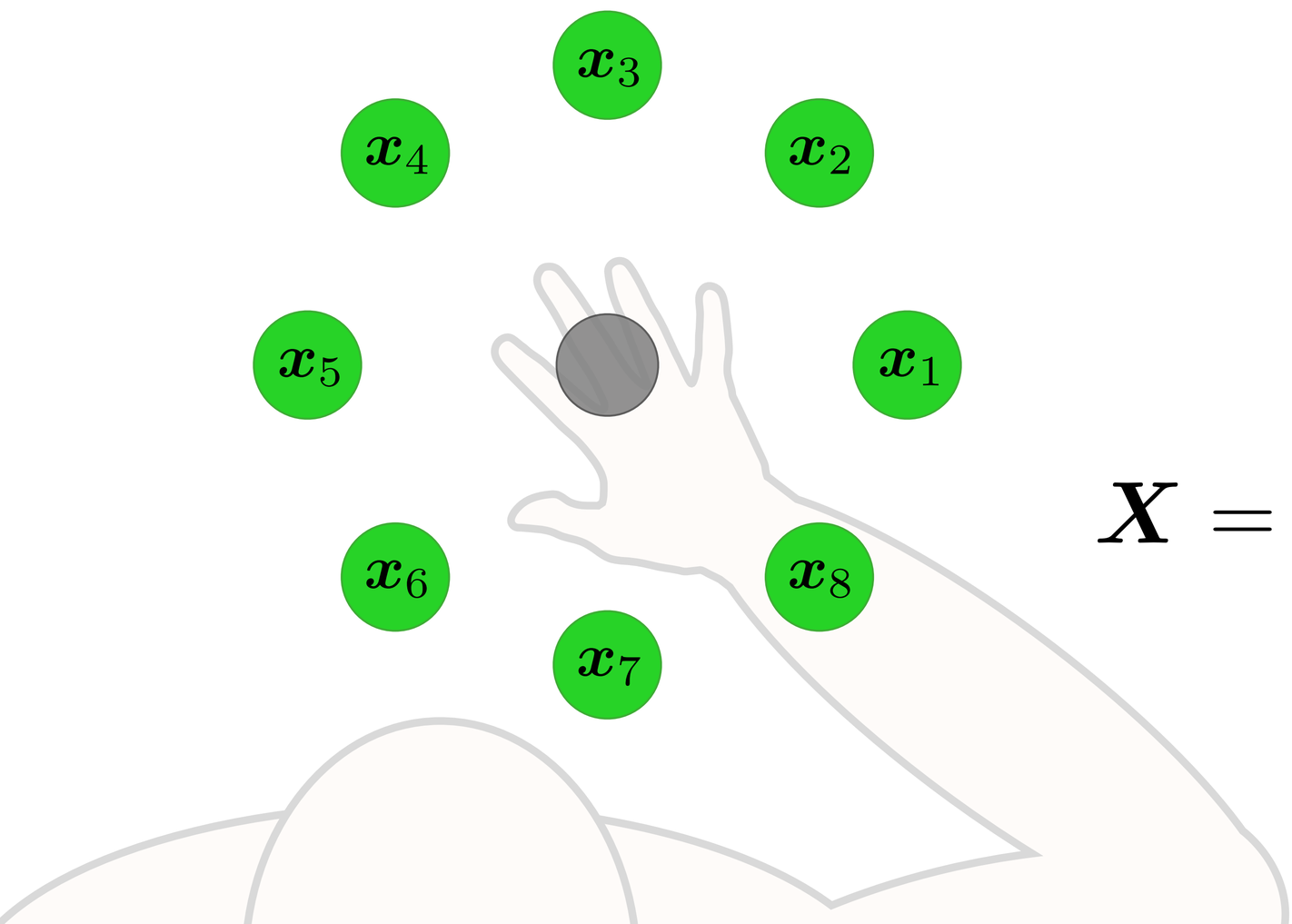

0.4
0.35
0.3
0.25
0.2
y (m)
90°
0°
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
x (m)


6
6


$\theta_s = 30°$
$\overline{\Delta\theta}_s = -3.7°$
$\sigma_{\theta_s} = 5.7°$
$\theta_s = 50°$
$\overline{\Delta\theta}_s = -7.2°$
$\sigma_{\theta_s} = 4.9°$
$\theta_e = 70°$
$\overline{\Delta\theta}_e = 9.3°$
$\sigma_{\theta_e} = 4.6°$
$\theta_e = 110°$
$\overline{\Delta\theta}_e = 0.3°$
$\sigma_{\theta_e} = 4.1°$
0.6
0.4
0.2
0
-0.4
-0.2
0
0.2
0.4
y
x (m)

task-space acuity $= h\left(\overline{\Delta \boldsymbol{x}}, \Sigma_{\boldsymbol{x}}\right)$

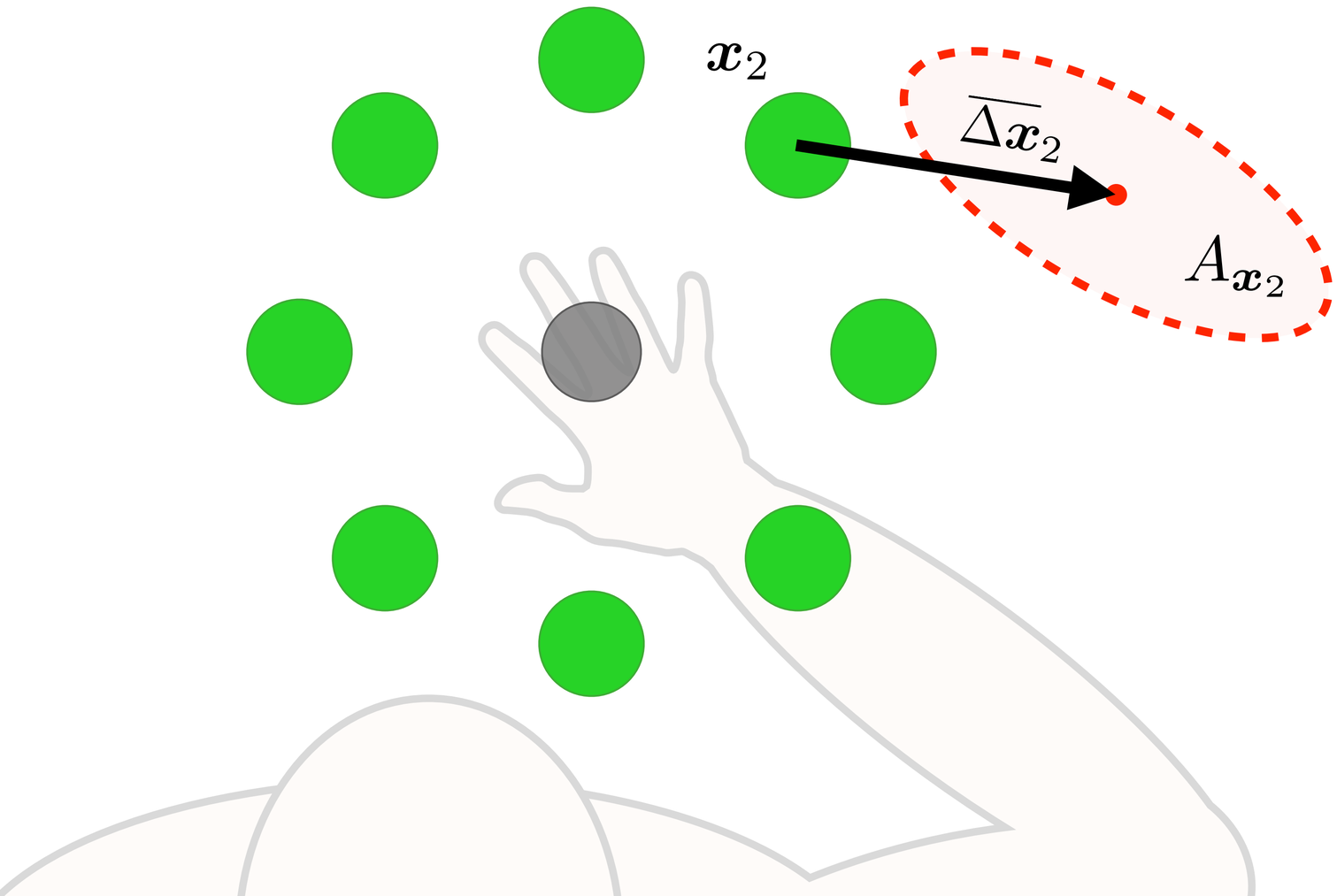
$$= \left(\frac{1}{8} \sum_{\boldsymbol{x} \in \boldsymbol{X}} A_{\boldsymbol{x}} \left(1 + \frac{\|\overline{\Delta \boldsymbol{x}}\|}{d}\right)\right)^{-1}$$



$$\boldsymbol{X} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_8\}$$

task-space acuity $= h\left(\overline{\Delta \boldsymbol{x}}, \Sigma_{\boldsymbol{x}}\right)$

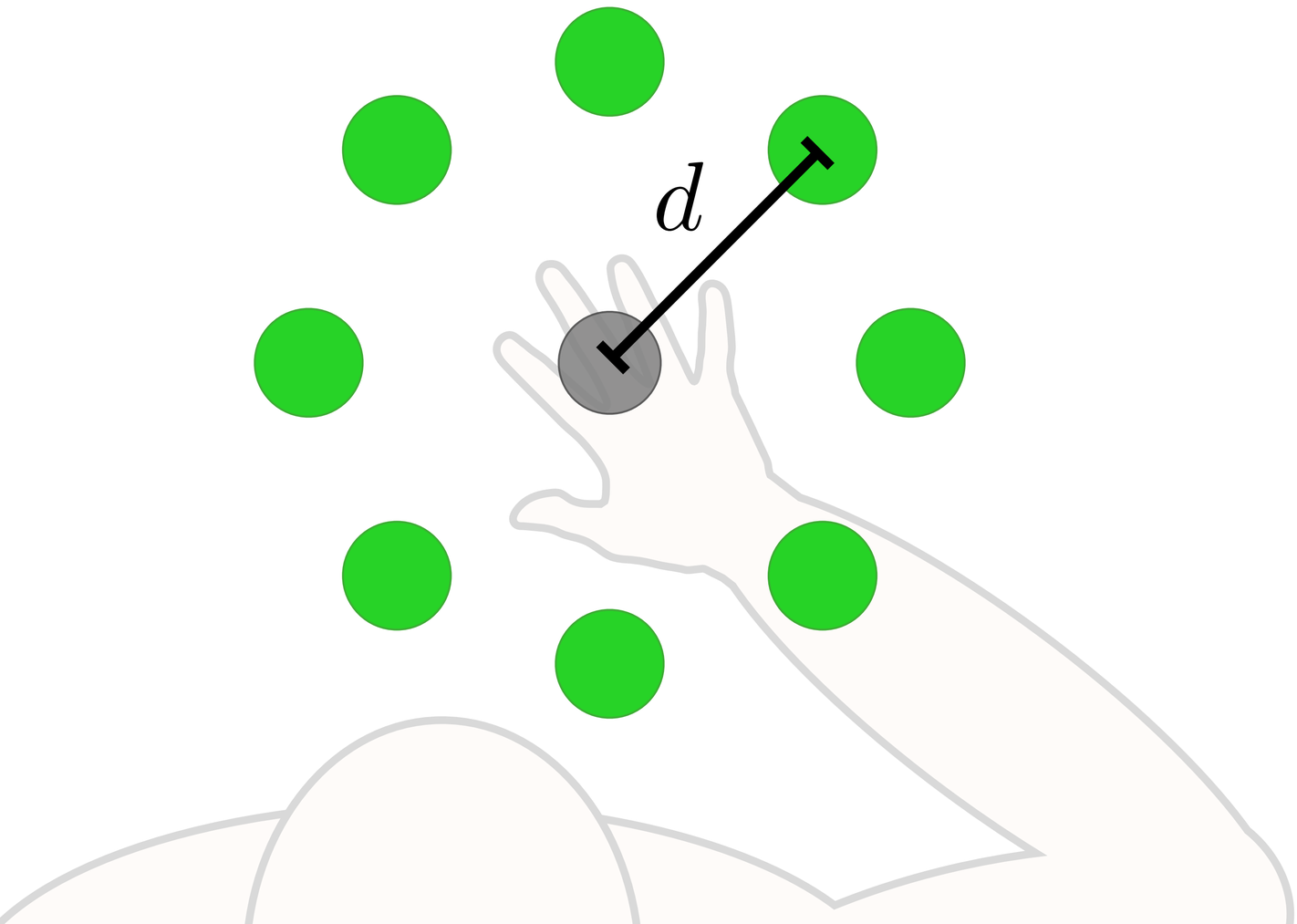
$$= \left(\frac{1}{8} \sum_{\boldsymbol{x} \in \boldsymbol{X}} A_{\boldsymbol{x}} \left(1 + \frac{\|\overline{\Delta \boldsymbol{x}}\|}{d}\right)\right)^{-1}$$



A = ellipse area

task-space acuity $= h\left(\overline{\Delta \boldsymbol{x}}, \Sigma_{\boldsymbol{x}}\right)$

$$= \left(\frac{1}{8} \sum_{\boldsymbol{x} \in \boldsymbol{X}} A_{\boldsymbol{x}} \left(1 + \frac{\|\overline{\Delta \boldsymbol{x}}\|}{d}\right)\right)^{-1}$$



joint-space acuity $= g\left(\overline{\Delta\theta}, \sigma_\theta\right)$

$$= \left(\frac{1}{2}\sum_{j\in J}\left(\frac{1}{2}\sum_{i\in\Theta_j}\sigma_{\theta_{ji}}\left(1+\frac{\left|\overline{\Delta\theta}_{ji}\right|}{i}\right)\right)\right)^{-1}$$

elbow
e

shoulder
s

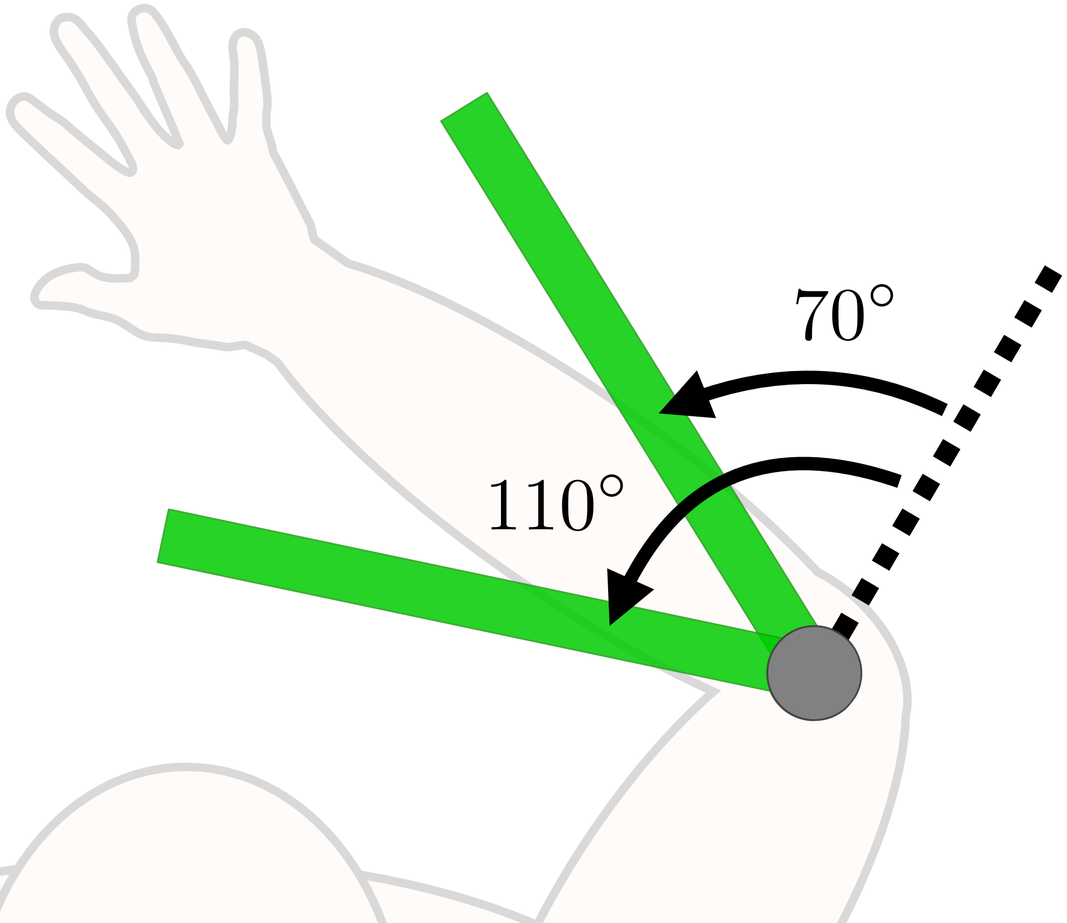
$$J = \{s, e\}$$

$$\Theta_e = \{70°, 110°\}$$

$$\Theta_s = \{30°, 50°\}$$

joint-space acuity $= g\left(\overline{\Delta\theta}, \sigma_\theta\right)$

$$= \left(\frac{1}{2}\sum_{j\in J}\left(\frac{1}{2}\sum_{i\in\Theta_j}\sigma_{\theta_{ji}}\left(1+\frac{\left|\overline{\Delta\theta}_{ji}\right|}{i}\right)\right)\right)^{-1}$$

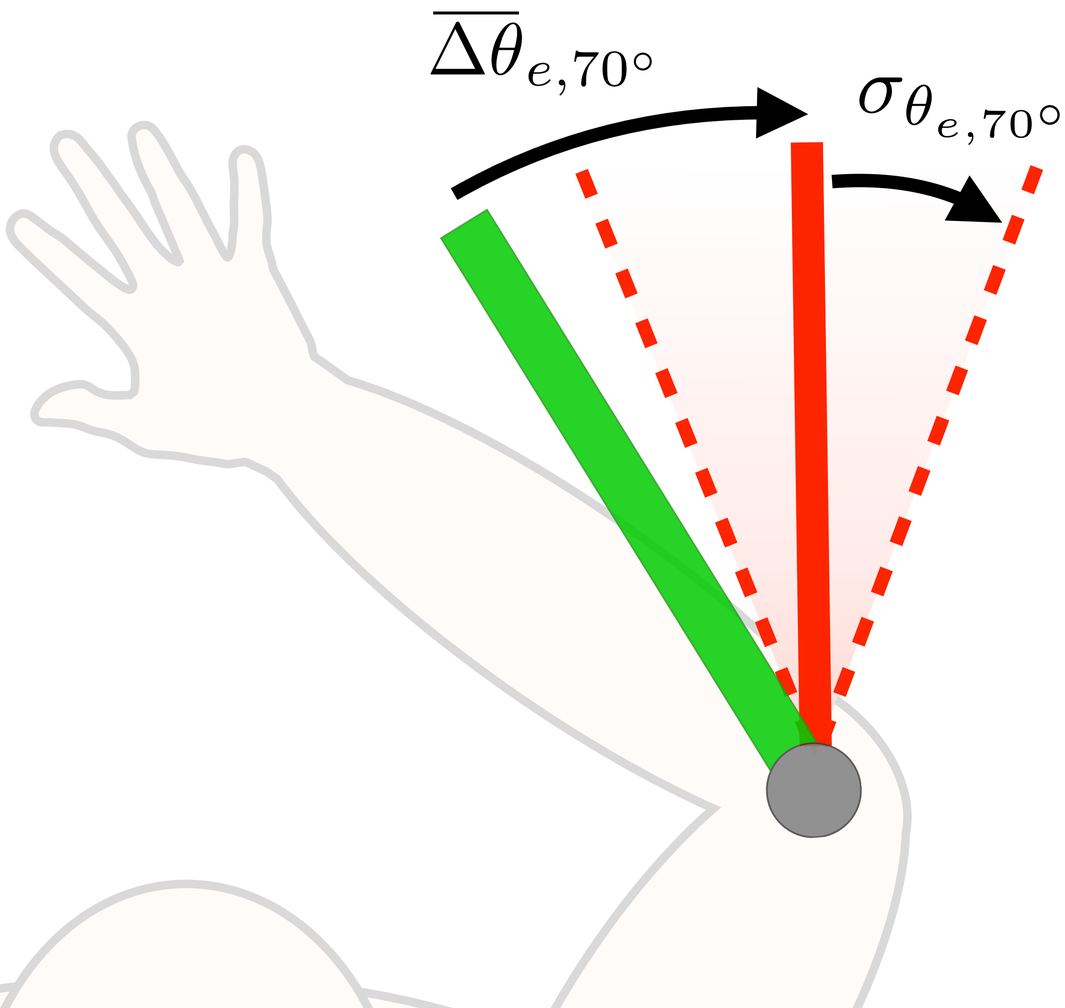


$$J = \{s, e\}$$

$$\Theta_e = \{70°, 110°\}$$

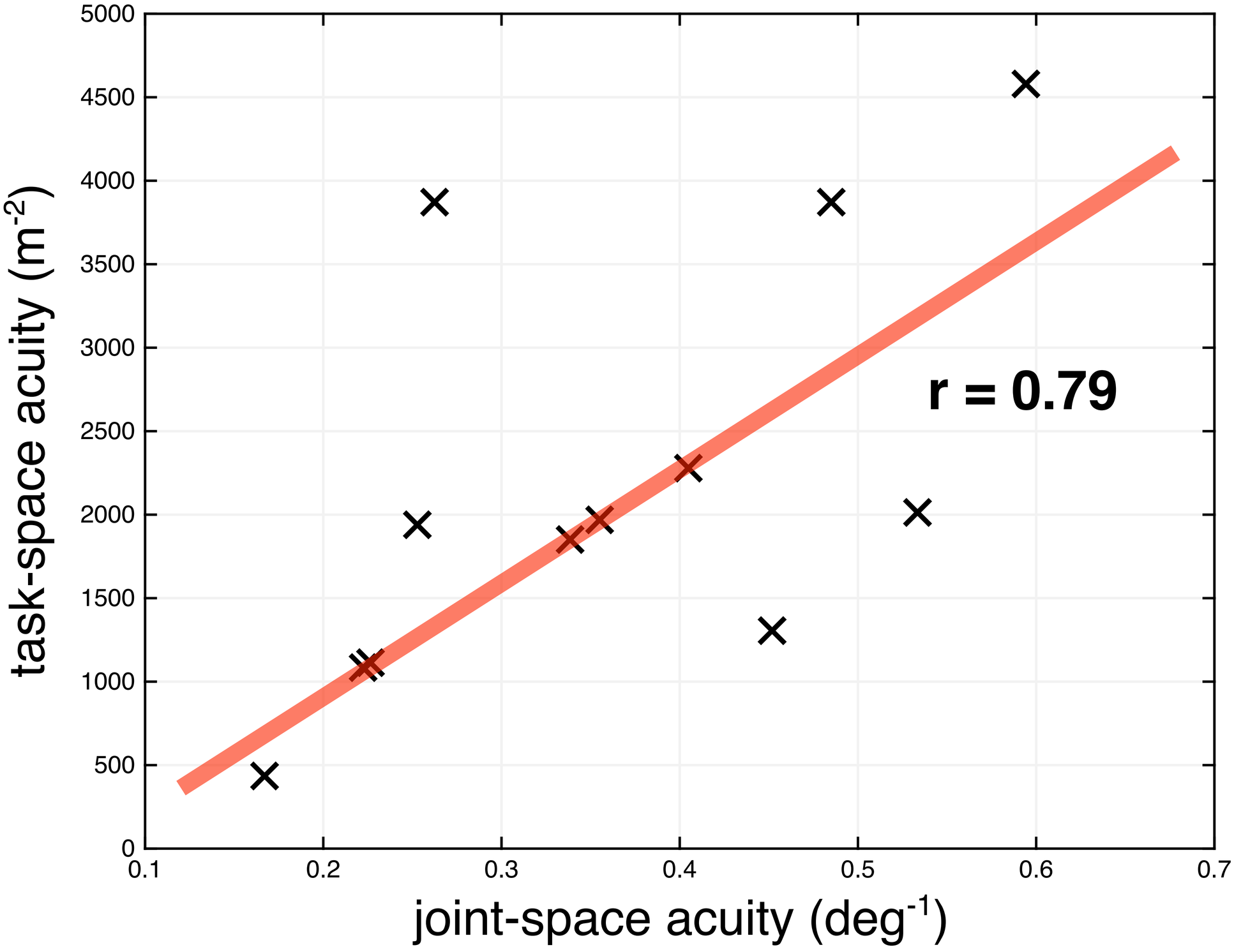
joint-space acuity $= g\left(\overline{\Delta\theta}, \sigma_\theta\right)$

$$= \left(\frac{1}{2}\sum_{j\in J}\left(\frac{1}{2}\sum_{i\in\Theta_j}\sigma_{\theta_{ji}}\left(1+\frac{|\overline{\Delta\theta}_{ji}|}{i}\right)\right)\right)^{-1}$$



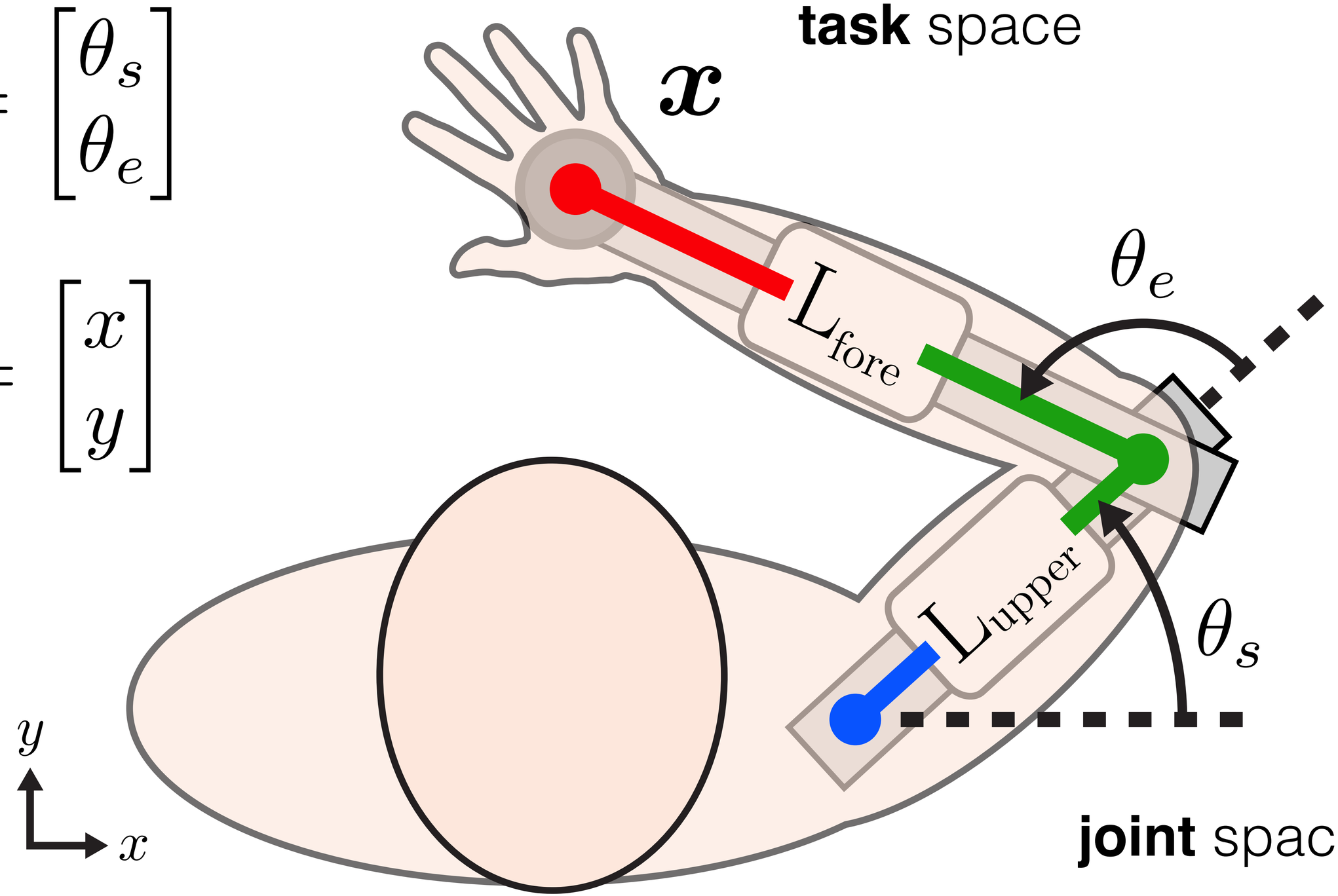
$$J = \{s, e\}$$

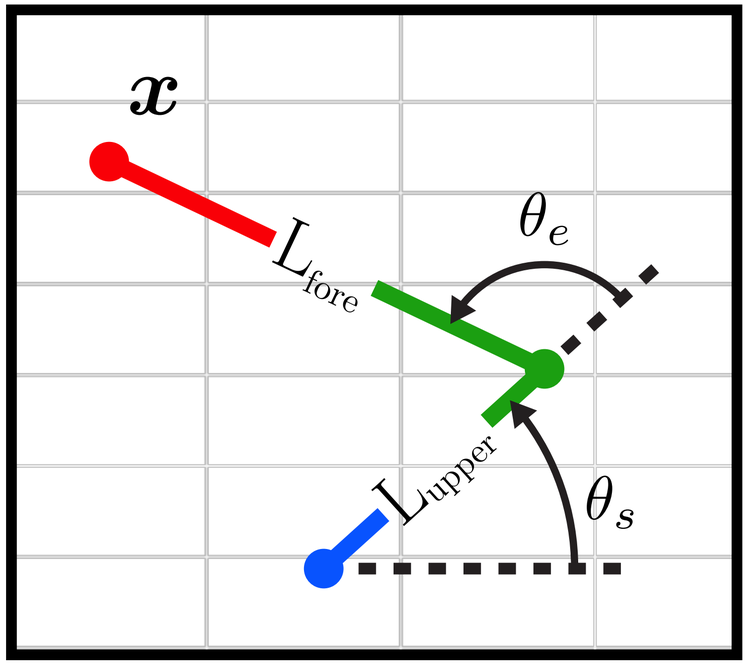
$$\Theta_e = \{70°, 110°\}$$


5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
task-space acuity (m-2)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
joint-space acuity (deg-1)
r = 0.79

$$\boldsymbol{\theta} = \begin{bmatrix} \theta_s \\ \theta_e \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

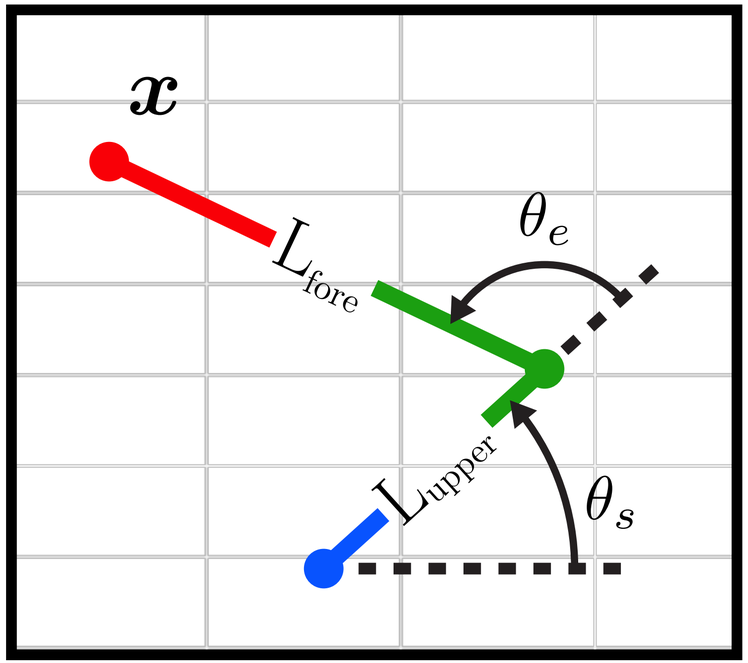




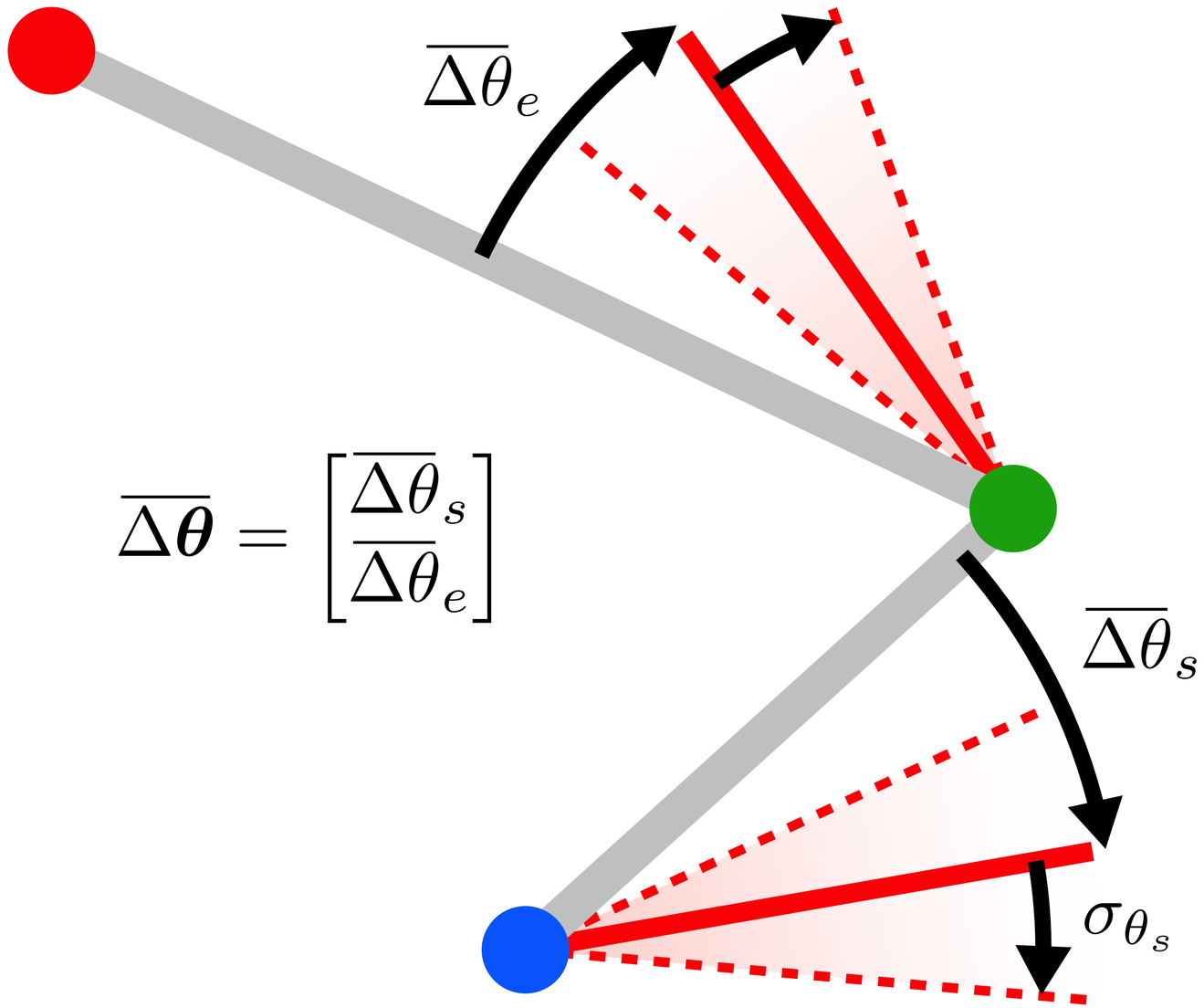
forward kinematics

$$\boldsymbol{x} = f(\boldsymbol{\theta})$$

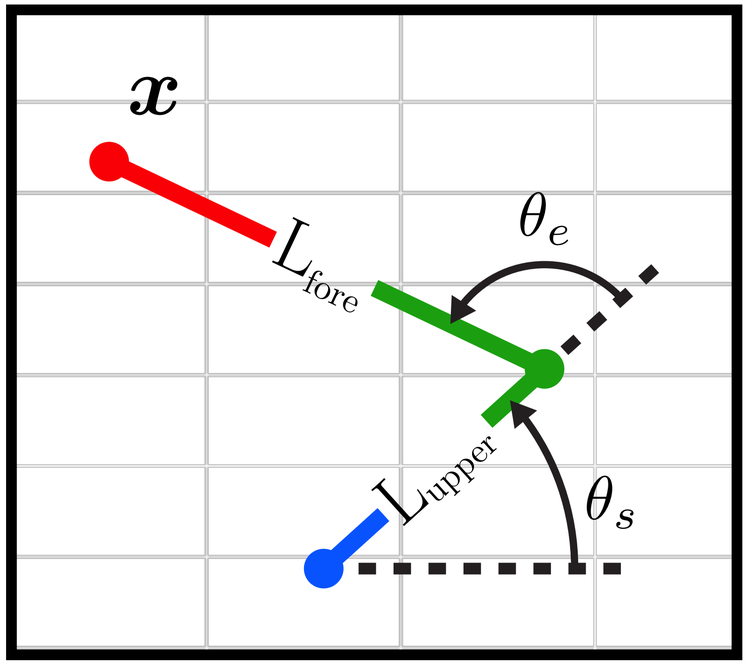
$$= \begin{bmatrix} \mathrm{L}_{\mathrm{upper}} \cos(\theta_s) + \mathrm{L}_{\mathrm{fore}} \cos(\theta_s + \theta_e) \\ \mathrm{L}_{\mathrm{upper}} \sin(\theta_s) + \mathrm{L}_{\mathrm{fore}} \sin(\theta_s + \theta_e) \end{bmatrix}$$



$$\Sigma_{\boldsymbol{\theta}} = \begin{bmatrix} \sigma^2_{\theta_s} & 0 \\ 0 & \sigma^2_{\theta_e} \end{bmatrix}$$



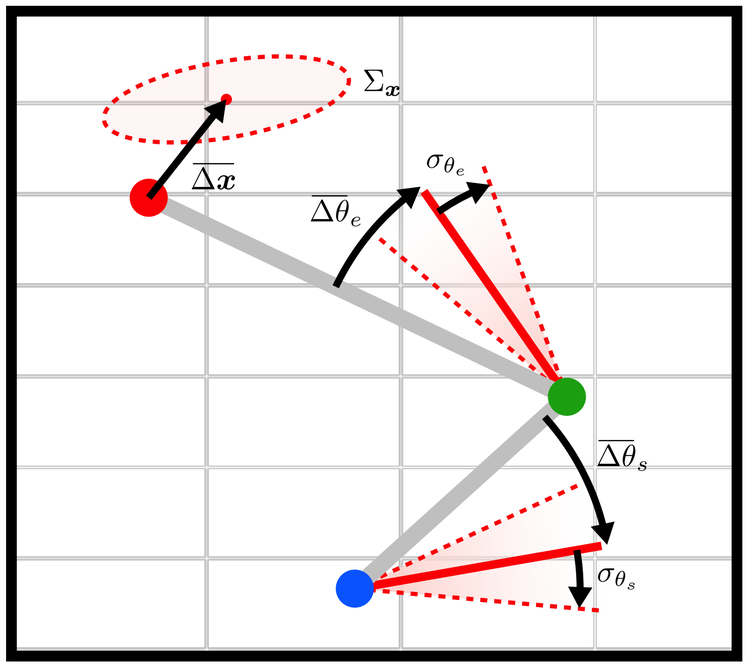
$$\overline{\Delta\boldsymbol{\theta}} = \begin{bmatrix} \overline{\Delta\theta}_s \\ \overline{\Delta\theta}_e \end{bmatrix}$$



forward kinematics

$$\boldsymbol{x} = f(\boldsymbol{\theta})$$

$$= \begin{bmatrix} \mathrm{L}_{\mathrm{upper}} \cos(\theta_s) + \mathrm{L}_{\mathrm{fore}} \cos(\theta_s + \theta_e) \\ \mathrm{L}_{\mathrm{upper}} \sin(\theta_s) + \mathrm{L}_{\mathrm{fore}} \sin(\theta_s + \theta_e) \end{bmatrix}$$



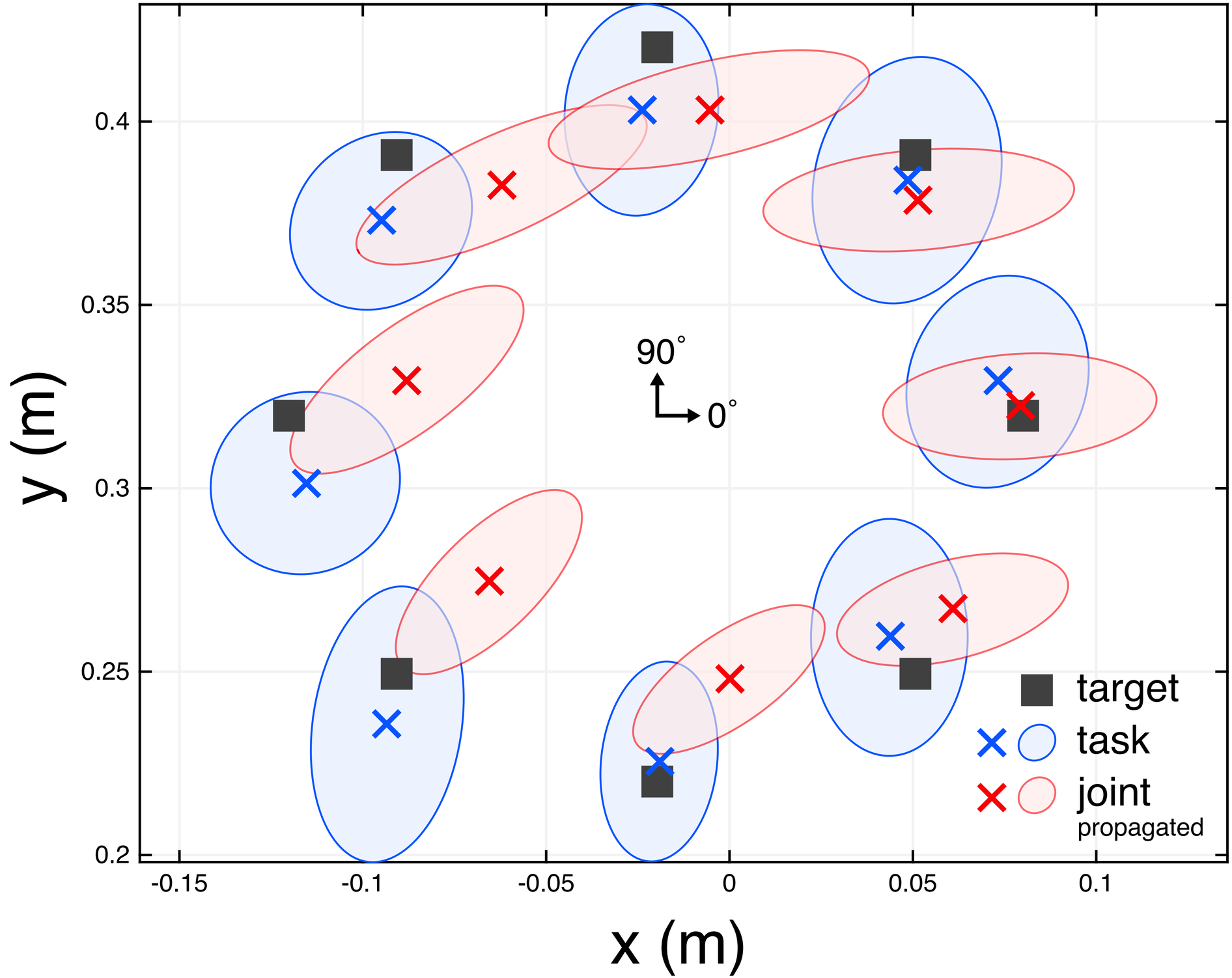
error propagation

$$\overline{\Delta \boldsymbol{x}} = f(\boldsymbol{\theta} + \overline{\Delta \boldsymbol{\theta}}) - \boldsymbol{x}$$

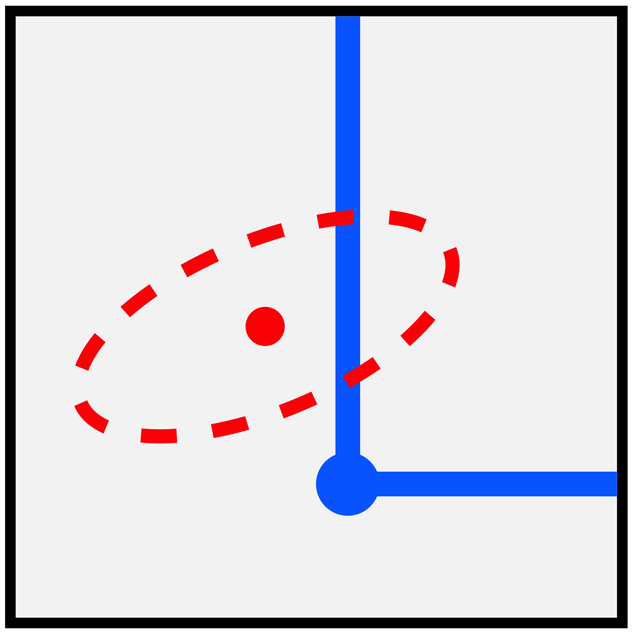
$$\Sigma_{\boldsymbol{x}} = J \Sigma_{\boldsymbol{\theta}} J^T$$

where

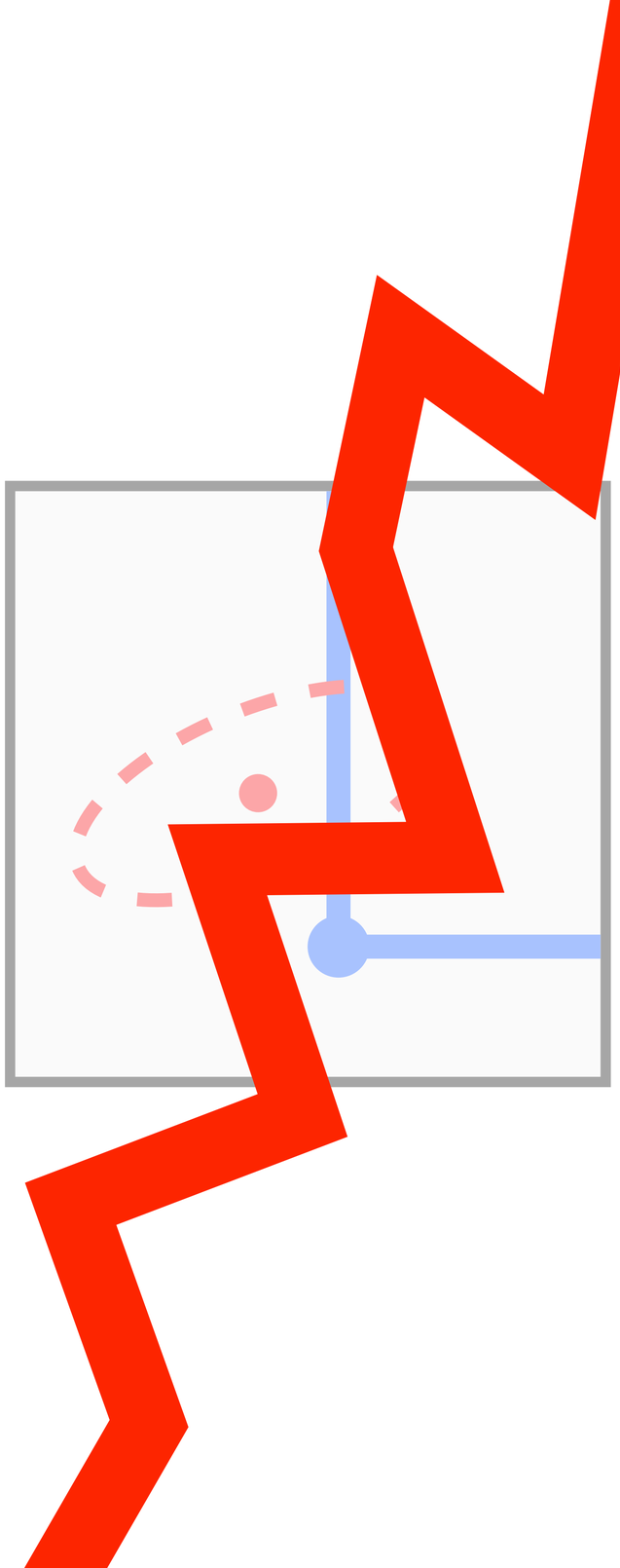
$$\boldsymbol{\theta} = f^{-1}(\boldsymbol{x})$$


0.4
0.35
0.3
0.25
0.2
y (m)
-0.15
-0.1
-0.05
0
0.05
0.1
x (m)
90°
0°
target
task
joint
propagated

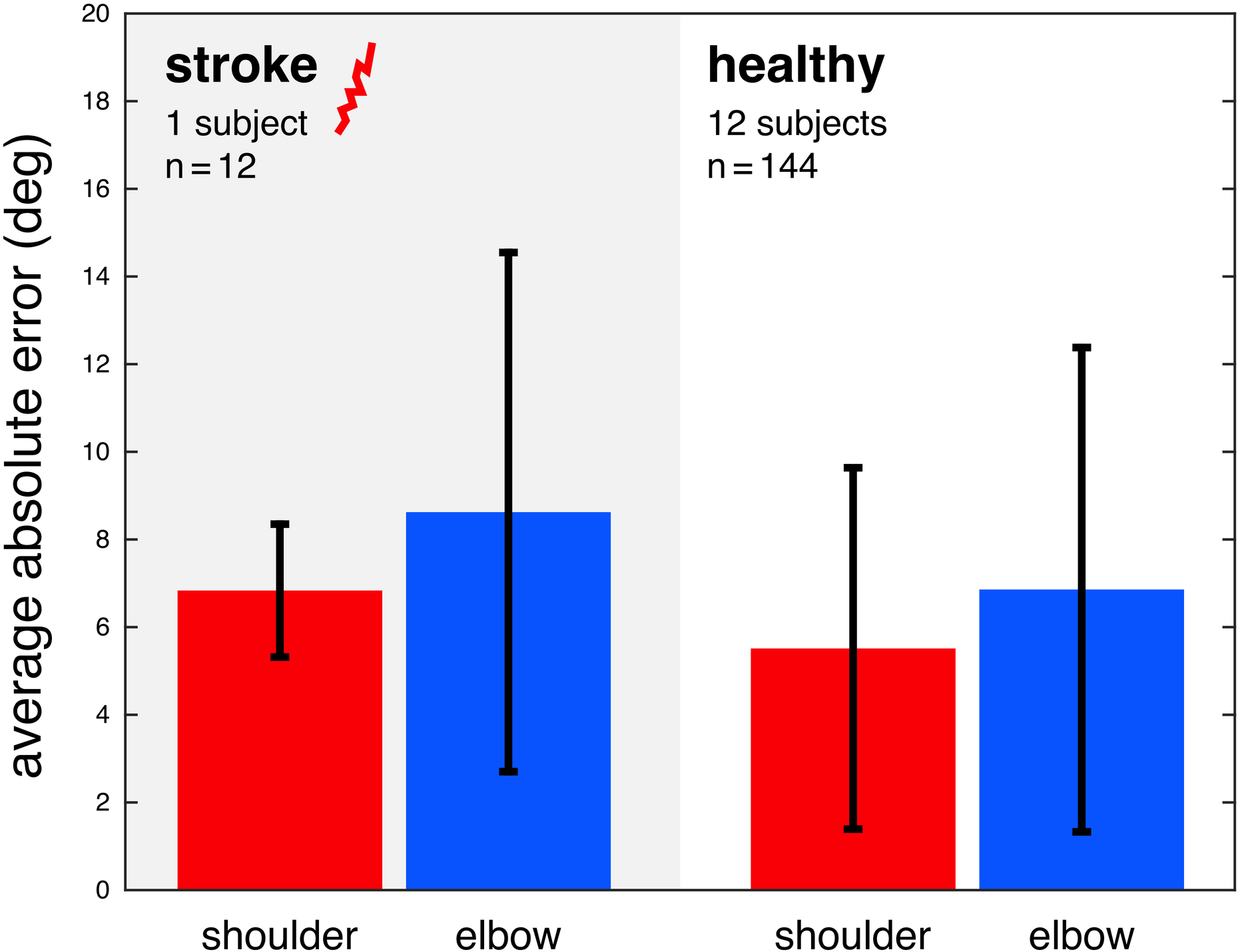
takeaways



- **strong correlation** in overall acuity
- **greater accuracy in task space**, in line with neuroscientific evidence
- **precision differences in direction**, not magnitude



?


stroke
1 subject
n = 12
healthy
12 subjects
n = 144
average absolute error (deg)
0
2
4
6
8
10
12
14
16
18
20
shoulder
elbow
shoulder
elbow

Thanks

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